Testing Inequality Restrictions in Multifactor Asset-Pricing Models

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Abstract

We develop an inequality constraints testing framework to assess the consistency of several multifactor models with the time-series and cross-sectional restrictions imposed by the intertemporal CAPM (ICAPM). Our tests of joint sign restrictions take into account the estimation error in the model parameters as well as the uncertainty arising from potential model misspecification. With a few exceptions, we cannot reject the null of consistency of the considered models with the ICAPM restrictions when using size and book-to-market, and size and momentum sorted portfolios as test assets. As argued by Fama (1991), the ICAPM may be a "fishing license" after all.

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1. Introduction

Multifactor asset-pricing models seek to explain cross-sectional differences in expected asset returns in terms of exposures to one or more sources of systematic risk. The capital asset pricing model (CAPM) of Treynor (1961), Sharpe (1964), Lintner (1965), and Mossin (1966) is the cornerstone of modern asset-pricing theory. It posits that the expected return on an asset is proportional to its covariance with the return on aggregate market wealth. The CAPM is a single-period model which, as shown by Fama (1970), can be treated as if it holds intertemporally only if the preferences and future investment opportunities are constant. However, as shown by Merton (1973), the CAPM does not hold in an intertemporal setting when the investor faces a state-dependent investment opportunity set.

The intertemporal CAPM (ICAPM) of Merton (1973) extends the CAPM to a multi-period framework. Unlike the single-period maximizer of the CAPM who does not take into account events beyond the current period, the intertemporal maximizer of the ICAPM also takes into account the relationship between current returns and returns that will be available in the future. This gives rise to additional sources of risk that an investor has to hedge against. According to the ICAPM, the expected return on an asset is not only proportional to the asset's covariance with the market portfolio return, but also to the asset's covariance with changes in the investment opportunity set. Following Cochrane (2005), the cross-sectional equilibrium relation between expected return and risk in the context of the ICAPM can be expressed as follows:

$$E_t(R_{i,t+1}) - R_{f,t} = \lambda \text{Cov}_t(R_{i,t+1}, R_{m,t+1}) + \lambda_z \text{Cov}_t(R_{i,t+1}, \Delta z_{t+1}), \tag{1}$$

where R_i denotes the expected return on asset i, R_f denotes the risk-free rate, R_m is the return on aggregate market wealth or simply the market portfolio, λ is the market price of covariance risk (which corresponds to the coefficient of relative risk aversion of the representative investor), λ_z is the intertemporal price of covariance risk, and Δz denotes innovations in state variables that capture uncertainty about future investment opportunities.

The second term of equation (1) is the expected return component that arises as compensation for unexpected changes in the investment opportunity set. These changes in investment opportunities are captured by the state variables z, which are essentially variables that describe the conditional distribution of returns that will be available in the future. The fact that the ICAPM does not explicitly identify these state variables has prompted Fama (1991) to label it a “fishing license”, in the sense that it essentially allows applied researchers to choose from a wide range of potential risk factors and use the ICAPM as a theoretical justification for relatively ad-hoc empirical specifications. Although the ICAPM does not explicitly tell us what the state variables are, there are several restrictions that candidate state variables need to satisfy for their innovations to be considered candidate risk factors in an ICAPM setting (Maio and Santa-Clara, 2012). First, the candidate state variables should predict changes in the investment opportunity set. Second, if a state variable predicts positive (negative) changes in investment opportunities in the time-series, then its innovation should earn a positive (negative) intertemporal price of covariance risk in the cross-sectional relation. Finally, the market price of risk should be an economically plausible estimate of the coefficient of relative risk aversion of the representative investor.

Therefore, it would seem that the ICAPM cannot be used as a theoretical justification for any multifactor model as it imposes several restrictions on the time-series and cross-sectional behavior of the candidate state variables and their innovations. However, the mere existence of these theoretical restrictions would not bear much weight against the claim that the ICAPM is a “fishing license” if these restrictions were indeed rarely violated in practice. Current research provides mixed evidence as to whether the ICAPM restrictions are satisfied in empirical tests of multifactor asset-pricing models. Maio and Santa-Clara (2012) consider eight popular multifactor asset-pricing specifications and find that most of these models are not consistent with an ICAPM interpretation. Lutzenberger (2015) replicates the Maio and Santa-Clara (2012) study for the European stock market and reaches similar conclusions. On the other hand, Boons (2016) focuses on state variables that forecast macroeconomic activity and the prices of covariance risk that the innovations in these state variables earn in a large cross-section of individual stocks. He finds consistency of the considered models with the ICAPM. Cooper and Maio (2018) study traded factor models and focus in particular on a number of recent prominent models incorporating investment and profitability factors. They find that the models studied are ”to a large degree compatible with the ICAPM framework” but none of them satisfies all the restrictions imposed by the ICAPM. Barroso, Boons and Karehnke (2019) focus on non-traded factor models, and account for time-variation in the risk premia by analyzing the conditional asset-pricing implications of the ICAPM. They find that conditional risk premia in a large cross-section of individual stocks are consistent in sign with how the state variables predict consumption growth in the time-series. Despite the lack of consensus, what all these studies have in common is the fact that the consistency of the models with the restrictions imposed by the ICAPM is assessed through a visual exercise whereby the researcher compares the signs of the slope estimates in the predictive regressions with the signs of the price of covariance risk estimates in the cross-sectional regressions.

We present a rigorous econometric framework to formally evaluate the consistency of a multifactor model with the time-series and cross-sectional restrictions imposed by the ICAPM and provide an in-depth empirical analysis to demonstrate the relevance of our methodological results. We focus on the empirical performance of nine multifactor models using two different sets of test assets and different estimation methods. First, we run multiple predictive ordinary least squares (OLS) time-series regressions to estimate the slope coefficients associated with the state variables. This allows us to obtain an *a priori* knowledge of the sign restrictions that the prices of covariance risk must satisfy for the various multifactor models to receive an ICAPM interpretation. Second, we estimate the prices of covariance risk by running two-pass cross-sectional regressions of average realized excess returns on the estimated covariances between the test asset returns and the innovations in the state variables (see Kan, Robotti, and Shanken (2013)). The estimation of the prices of covariance risk is performed using OLS, generalized least squares (GLS), and weighted least squares (WLS) weighting schemes. Third, we develop and implement a multivariate inequality test, based on Wolak (1987, 1989), to determine whether the signs of the prices of covariance risk are consistent with the signs of the slope coefficients in the predictive regressions. This allows us to go beyond the common practice of informally comparing the signs of the estimated coefficients in the predictive regressions with the signs of the estimated prices of covariance risk in the cross-sectional regressions. Our methods account for the estimation error in the covariances and for the fact that the consistency of a multifactor model with the implications of the ICAPM should be evaluated using tests of joint sign restrictions across factors. Importantly, in the estimation of the prices of covariance risk and in the tests of the sign restrictions, we employ asymptotic standard errors that are robust to potential model misspecification in addition to the traditional standard errors computed under the assumption that the model is correctly specified.

Our testing methodology delivers conclusions that are substantially different from the ones reached by following the common practice of visually comparing sign estimates that is used in the existing literature. If we simply compare the signs of the estimates, we find sign consistency in 20 out of 54 cases, which suggests that most multifactor models do not satisfy the restrictions imposed by the ICAPM. However, if we apply our multivariate inequality test, we find that in 43 out of 54 cases we do not have enough evidence to reject the null hypothesis of sign consistency at the 5% level, which indicates that most models do satisfy the ICAPM restrictions. Another important finding is that accounting for potential model misspecification can make a significant difference in terms of the conclusions reached. When the test statistic is computed using the traditional Fama and MacBeth (1973) standard errors, we obtain sign consistency in 32 out of 54 cases, but when misspecification-robust standard errors are used, the sign restrictions are satisfied in 43 out of 54 cases. Moreover, we find that the use of misspecification-robust standard errors makes a substantial difference when the correlation between the returns on the test assets and the factors is low, as it is the case when using size and momentum sorted portfolios (see Kan, Robotti, and Shanken (2013) for a discussion of this point). Specifically, when the 25 size and momentum sorted portfolios are used as test assets, the sign consistency hypothesis is rejected in 17 out of 27 cases if the test statistics are computed using Fama and MacBeth (1973) standard errors but only in 8 out of 27 cases if misspecification-robust standard errors are used. On the other hand, when the test assets are the 25 size and book-to-market sorted portfolios, the test statistics based on the Fama and MacBeth (1973) asymptotic variance indicate rejection of the null in only 5 out of 27 cases, whereas the misspecification-robust test statistics indicate rejection in 3 out of 27 cases.

The rest of the paper is organized as follows. Section 2 presents an asymptotic analysis of the estimates of the prices of covariance risk under potentially misspecified models. In addition, we provide the limiting distribution of the sample cross-sectional R^2. Finally, we develop a multiple sign restriction test and show how this test accounts for estimation and model misspecification uncertainty. Section 3 presents our main empirical findings and Section 4 concludes. The proofs of the propositions are provided in the Appendix.

2. Asymptotic analysis under potentially misspecified models

As discussed in the introduction, an asset-pricing model seeks to explain cross-sectional differences in expected asset returns in terms of asset exposures computed relative to the model's systematic economic factors. The two-pass cross-sectional regression (CSR) methodology has become the most popular approach for estimating and testing linear asset-pricing models. Despite the existence of many variations of the CSR methodology, the basic approach always involves two steps or passes. In the first pass, the betas of the test assets are estimated from OLS time-series regressions of returns on some common factors. In the second pass, the returns on the test assets are regressed on the betas estimated from the first pass. The intercept and the slope coeficients from the second-pass CSR are the estimates of the zero-beta rate and factor risk premia.

Let f be a K-vector of factors and R a vector of excess returns (i.e., returns on zero investment portfolios) on N test assets. We define $Y = [f', \ R']'$ and its mean and covariance matrix as

$$\mu = E[Y] \equiv \begin{bmatrix} \mu_f \\ \mu_R \end{bmatrix}, \tag{2}$$

$$V = \mathrm{Var}[Y] \equiv \begin{bmatrix} V_f & V_{f,R} \\ V_{R,f} & V_R \end{bmatrix}, \tag{3}$$

where V is assumed to be positive definite. The multiple regression betas of the N assets with respect to the K factors are defined as $\beta = V_{R,f}V_f^{-1}$. These are measures of systematic risk or the sensitivity of the asset returns to the factors. In addition, we denote the covariance matrix of the residuals of the N assets by $\Sigma = V_R - V_{R,f}V_f^{-1}V_{f,R}$.

In the following analysis, we focus on an excess returns specification of the CSR methodology. This essentially involves constraining the zero-beta rate to equal the risk-free rate, a practice that is common in other parts of the empirical asset-pricing literature. For example, studies that focus on time-series "alphas" when all factors are traded impose this restriction (see, for example, Gibbons, Ross, and Shanken (1989)). We implement the zero-beta rate restriction in the CSR context by working with test asset returns in excess of the T-bill rate, while excluding the constant from the expected return relations. Thus, the proposed K-factor beta-pricing model specifies that asset expected excess returns are linear in the betas, i.e.,

$$\mu_R = \beta\gamma, \tag{4}$$

where β is assumed to be of full column rank and γ is a vector consisting of the risk premia on the K factors. When the model is misspecified, the pricing-error vector, $\mu_R - \beta\gamma$, will be nonzero for all values of γ. In that case, it makes sense to choose γ to minimize some aggregation of pricing errors. Denoting by W an $N \times N$ symmetric positive-definite weighting matrix, we define the (pseudo) risk premia as the choice of γ that minimizes the quadratic form of pricing errors:

$$\gamma_W = \text{argmin}_\gamma (\mu_R - \beta\gamma)' W (\mu_R - \beta\gamma) = (\beta' W \beta)^{-1} \beta' W \mu_R. \tag{5}$$

The corresponding pricing errors of the N assets are then given by

$$e_W = \mu_R - \beta\gamma_W = [I_N - \beta(\beta' W \beta)^{-1} \beta' W] \mu_R. \tag{6}$$

In addition to the pricing errors, researchers are often interested in a normalized goodness-of-fit measure for a model. A popular measure is the cross-sectional R^2. Following Kandel and Stambaugh (1995), this is defined as

$$\rho_W^2 = 1 - \frac{Q}{Q_0}, \tag{7}$$

where

$$Q_0 = \mu_R' W \mu_R, \tag{8}$$

$$Q = e_W' W e_W = \mu_R' W \mu_R - \mu_R' W \beta(\beta' W \beta)^{-1} \beta' W \mu_R. \tag{9}$$

Note that $0 \leq \rho_W^2 \leq 1$ and it is a decreasing function of the aggregate pricing errors $Q = e_W' W e_W$. Thus, ρ_W^2 is a natural measure of goodness of fit.

While the betas are typically used as the regressors in the second-pass CSR, there is a potential issue with the use of multiple regression betas when $K > 1$: in general, the beta of an asset with respect to a particular factor depends on what other factors are included in the first-pass time-series OLS regression. As a consequence, the interpretation of the risk premia γ in the context of model selection becomes problematic. To overcome this problem, in the subsequent analysis we focus on an alternative second-pass CSR that uses the covariances $V_{R,f}$ instead of the betas β as the regressors.[1] Let λ_W be the choice of coefficients that minimizes the quadratic form of pricing errors:

$$\lambda_W = \text{argmin}_\lambda (\mu_R - V_{R,f}\lambda)' W (\mu_R - V_{R,f}\lambda) = (V_{f,R} W V_{R,f})^{-1} V_{f,R} W \mu_R. \tag{10}$$

Given (5) and (10), there is a one-to-one correspondence between γ_W and λ_W:

$$\lambda_W = V_f^{-1} \gamma_W. \tag{11}$$

[1] Another solution to this problem is to use simple regression betas as the regressors in the second-pass CSR, as in Chen, Roll, and Ross (1986) and Jagannathan and Wang (1996, 1998). Kan and Robotti (2011) provide asymptotic results for the CSR with simple regression betas under potentially misspecified models.

It is easy to see that the pricing errors from this alternative second-pass CSR, $e_W = \mu_R - V_{R,f}\lambda_W$, are the same as those in (6). It follows that the ρ_W^2 for these two CSRs are also identical. However, it is important to note that unless V_f is a diagonal matrix, $\lambda_{W,i} = 0$ does not imply $\gamma_{W,i} = 0$, and vice versa (see Kan, Robotti, and Shanken, 2013, for a detailed discussion of this point).

It should be emphasized that unless the model is correctly specified, γ_W, λ_W, e_W, and ρ_W^2 depend on the choice of W. Popular choices of W in the literature are $W = I_N$ (OLS CSR), $W = V_R^{-1}$ (GLS CSR), and $W = \Sigma_d^{-1}$ (WLS CSR), where $\Sigma_d = \text{Diag}(\Sigma)$. To simplify the notation, we suppress the subscript W from γ_W, λ_W, e_W, and ρ_W^2 when the choice of W is clear from the context.

We now turn to estimation of the models. Let $Y_t = [f_t', \ R_t']'$, where f_t is the vector of K proposed factors at time t and R_t is the vector of N excess returns on the test assets at time t. We assume the time series Y_t is jointly stationary and ergodic, with finite fourth moment. Suppose we have T observations on Y_t and denote the sample moments of Y_t by

$$\hat{\mu} = \begin{bmatrix} \hat{\mu}_f \\ \hat{\mu}_R \end{bmatrix} = \frac{1}{T}\sum_{t=1}^{T} Y_t, \tag{12}$$

$$\hat{V} = \begin{bmatrix} \hat{V}_f & \hat{V}_{f,R} \\ \hat{V}_{R,f} & \hat{V}_R \end{bmatrix} = \frac{1}{T}\sum_{t=1}^{T}(Y_t - \hat{\mu})(Y_t - \hat{\mu})'. \tag{13}$$

When the weighting matrix W is known (say OLS CSR), we can estimate λ_W in (10) by

$$\hat{\lambda} = (\hat{V}_{f,R} W \hat{V}_{R,f})^{-1}\hat{V}_{f,R} W \hat{\mu}_R. \tag{14}$$

In the GLS and WLS cases, the weighting matrix W involves unknown parameters and, therefore, we need to substitute a consistent estimate of W, say $\hat{W}$, in (14). This is typically the corresponding matrix of sample moments, $\hat{W} = \hat{V}_R^{-1}$ for GLS and $\hat{W} = \text{Diag}(\hat{\Sigma})^{-1} = \hat{\Sigma}_d^{-1}$ for WLS, where $\hat{\Sigma} = \hat{V}_R - \hat{V}_{R,f}\hat{V}_f^{-1}\hat{V}_{f,R}$.

The sample measure of ρ^2 is similarly defined as

$$\hat{\rho}^2 = 1 - \frac{\hat{Q}}{\hat{Q}_0}, \tag{15}$$

where $\hat{Q}_0$ and $\hat{Q}$ are consistent estimators of Q_0 and Q in (8) and (9), respectively. When W is known, we estimate Q_0 and Q using

$$\hat{Q}_0 = \hat{\mu}_R' W \hat{\mu}_R, \tag{16}$$

$$\hat{Q} = \hat{\mu}_R' W \hat{\mu}_R - \hat{\mu}_R' W \hat{V}_{R,f} (\hat{V}_{f,R} W \hat{V}_{R,f})^{-1} \hat{V}_{f,R} W \hat{\mu}_R. \tag{17}$$

When W is not known, we replace W with $\hat{W}$.

2.1. Asymptotic distribution of $\hat{\boldsymbol{\lambda}}$ under potentially misspecified models

When computing the standard error of $\hat{\lambda}$, researchers typically rely on the asymptotic distribution of $\hat{\lambda}$ under the assumption that the model is correctly specified. In the following proposition, we relax this assumption and provide general expressions for the asymptotic variance of $\hat{\lambda}$ for the OLS, GLS, and WLS cases under potential model misspecification.

Proposition 1. *Under a potentially misspecified model, the asymptotic distribution of $\hat{\lambda}$ is given by*

$$\sqrt{T}(\hat{\lambda} - \lambda) \overset{A}{\sim} N(0_K, V(\hat{\lambda})), \tag{18}$$

where

$$V(\hat{\lambda}) = \sum_{j=-\infty}^{\infty} E[h_t h_{t+j}']. \tag{19}$$

To simplify the expressions for h_t, we define $G_t = V_{R,f} - (R_t - \mu_R)(f_t - \mu_f)'$, $H = (V_{f,R} W V_{R,f})^{-1}$, $A = HV_{f,R}W$, $\lambda_t = AR_t$, $u_t = e'W(R_t - \mu_R)$, and $\Psi_t = Diag(\epsilon_t \epsilon_t')$, where $\epsilon_t = R_t - \mu_R - \beta(f_t - \mu_f)$.

(a) With a known weighting matrix W, $\hat{\lambda} = (\hat{V}_{f,R} W \hat{V}_{R,f})^{-1} \hat{V}_{f,R} W \hat{\mu}_R$ and

$$h_t = (\lambda_t - \lambda) + AG_t\lambda + H(f_t - \mu_f)u_t. \tag{20}$$

(b) For GLS, $\hat{\lambda} = (\hat{V}_{f,R} \hat{V}_R^{-1} \hat{V}_{R,f})^{-1} \hat{V}_{f,R} \hat{V}_R^{-1} \hat{\mu}_R$ and

$$h_t = (\lambda_t - \lambda) + AG_t\lambda + H(f_t - \mu_f)u_t - (\lambda_t - \lambda)u_t. \tag{21}$$

(c) For WLS, $\hat{\lambda} = (\hat{V}_{f,R} \hat{\Sigma}_d^{-1} \hat{V}_{R,f})^{-1} \hat{V}_{f,R} \hat{\Sigma}_d^{-1} \hat{\mu}_R$ and

$$h_t = (\lambda_t - \lambda) + AG_t\lambda + H(f_t - \mu_f)u_t - A\Psi_t \Sigma_d^{-1} e. \tag{22}$$

When the model is correctly specified, we have:

$$h_t = (\lambda_t - \lambda) + AG_t\lambda. \tag{23}$$

Proof. See Appendix.

To conduct statistical tests, we need a consistent estimator of $V(\hat{\lambda})$. This can be obtained by replacing the h_t's with their sample counterparts $\hat{h}_t$'s. In particular, if h_t is uncorrelated over time, then we have $V(\hat{\lambda}) = E[h_t h_t']$, and its consistent estimator is given by

$$\hat{V}(\hat{\lambda}) = \frac{1}{T}\sum_{t=1}^{T} \hat{h}_t \hat{h}_t'. \tag{24}$$

When h_t is autocorrelated, one can use Newey and West's (1987) method to obtain a consistent estimator of $V(\hat{\lambda})$.

Inspection of (20) reveals that there are three sources of asymptotic variance for $\hat{\lambda}$. The first term $\lambda_t - \lambda$ measures the asymptotic variance of $\hat{\lambda}$ when the *true* covariances are used in the CSR. For example, if R_t is i.i.d., then λ_t is also i.i.d. and we can use the time-series variance of λ_t to compute the standard error of $\hat{\lambda}$. This coincides with the popular Fama and MacBeth (1973) method. Since the covariances are estimated with error, an errors-in-variables (EIV) problem is introduced in the second-pass CSR. The second term $AG_t\lambda$ is the EIV adjustment term that accounts for the estimation errors in the estimated covariances. The first two terms together give us the $V(\hat{\lambda})$ under the correctly specified model. When the model is misspecified ($e \neq 0_N$), there is a third term $H(f_t - \mu_f)u_t$, which we call the misspecification adjustment term. Traditionally, this term has been ignored by empirical researchers. Comparing (21) and (22) with the expression for h_t in (20), we see that there is an extra term in h_t associated with the use of $\hat{W}$ instead of W. This fourth term vanishes if the weighting matrix W is known.

2.2. Asymptotic distribution of the sample cross-sectional R^2

The sample R^2 ($\hat{\rho}^2$) in the second-pass CSR is a popular measure of goodness of fit for a model. A high $\hat{\rho}^2$ is viewed as evidence that the model under study does a good job of explaining the cross-section of expected returns. Lewellen, Nagel, and Shanken (2010) point out several pitfalls to using this approach and explore simulation techniques to obtain approximate confidence intervals for ρ^2.[2] In this subsection, we provide a formal statistical analysis of $\hat{\rho}^2$.

[2] Jagannathan, Kubota, and Takehara (1998), Kan and Zhang (1999), and Jagannathan and Wang (2007) use simulations to examine the sampling errors of the cross-sectional R^2 and risk premium estimates under the assumption that one of the factors is "useless," that is, independent of returns.

The asymptotic distribution of $\hat{\rho}^2$ crucially depends on the value of ρ^2. When $\rho^2 = 1$ (that is, a correctly specified model), the asymptotic distribution serves as the basis for a specification test of the asset-pricing model. This is an alternative to the various multivariate asset-pricing tests that have been developed in the literature. Although all of these tests focus on an aggregate pricing-error measure, the R^2-based test examines pricing errors in relation to the cross-sectional variation in expected returns, allowing for a simple and appealing interpretation. At the other extreme, the asymptotic distribution when $\rho^2 = 0$ (a misspecified model that does not explain any of the cross-sectional variation in expected returns) permits a test of whether the model has *any* explanatory power for expected returns.

When $0 < \rho^2 < 1$ (a misspecified model that provides some explanatory power), the case of primary interest, $\hat{\rho}^2$ is asymptotically normally distributed around its true value. It is readily verified that the asymptotic standard error of $\hat{\rho}^2$ approaches zero as $\rho^2 \to 0$ or $\rho^2 \to 1$, and thus it is not monotonic in ρ^2. The asymptotic normal distribution of $\hat{\rho}^2$ breaks down for the two extreme cases ($\rho^2 = 0$ or 1) because, by construction, $\hat{\rho}^2$ will always be above zero (even when $\rho^2 = 0$) and below one (even when $\rho^2 = 1$).

Proposition 2. *In the following, we set W to be V_R^{-1} and Σ_d^{-1} for the GLS and WLS cases, respectively.*

(a) When $\rho^2 = 1$,

$$T(\hat{\rho}^2 - 1) = -\frac{T\hat{Q}}{\hat{Q}_0} \overset{A}{\sim} -\sum_{j=1}^{N-K} \frac{\xi_j}{Q_0} x_j, \tag{25}$$

where the x_j's are independent χ_1^2 random variables, and the ξ_j's are the eigenvalues of

$$P' W^{\frac{1}{2}} S W^{\frac{1}{2}} P, \tag{26}$$

where P is an $N \times (N-K)$ orthonormal matrix with columns orthogonal to $W^{\frac{1}{2}} V_{R,f}$, S is the asymptotic covariance matrix of $\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \epsilon_t y_t$, and $y_t = 1 - \lambda'(f_t - \mu_f)$ is the normalized stochastic discount factor (SDF).

(b) When $0 < \rho^2 < 1$,

$$\sqrt{T}(\hat{\rho}^2 - \rho^2) \overset{A}{\sim} N\left(0, \sum_{j=-\infty}^{\infty} E[n_t n_{t+j}]\right), \tag{27}$$

where

$$n_t = 2\left[-u_t y_t + (1-\rho^2)v_t\right]/Q_0 \qquad \text{for known } W, \tag{28}$$

$$n_t = \left[u_t^2 - 2u_t y_t + (1-\rho^2)(2v_t - v_t^2)\right]/Q_0 \qquad \text{for } \hat{W} = \hat{V}_R^{-1}, \tag{29}$$

$$n_t = \left[-2u_t y_t + e'\Gamma_t e + (1-\rho^2)(2v_t - \mu_R'\Gamma_t\mu_R)\right]/Q_0 \qquad \text{for } \hat{W} = \hat{\Sigma}_d^{-1}, \tag{30}$$

with $\Gamma_t = \Sigma_d^{-1}\Psi_t\Sigma_d^{-1}$ *and* $v_t = \mu_R' W(R_t - \mu_R)$.

(c) When $\rho^2 = 0$,

$$T\hat{\rho}^2 \overset{A}{\sim} \sum_{j=1}^{K} \frac{\xi_j}{Q_0} x_j, \tag{31}$$

where the x_j*'s are independent* χ_1^2 *random variables and the* ξ_j*'s are the eigenvalues of*

$$(V_{f,R} W V_{R,f}) V(\hat{\lambda}), \tag{32}$$

where $V(\hat{\lambda})$ *is given in Proposition 1.*

Proof. See Appendix.

2.3. Multiple sign restriction test

In this section, we develop and implement a formal test of multiple sign restrictions. This is a multivariate inequality test based on results in the statistics literature due to Wolak (1987, 1989).

Suppose that interest lies in testing

$$H_0: \ \mathcal{Q}\lambda \geq 0_p \quad \text{vs.} \quad H_1: \ \lambda \in \Re^K, \tag{33}$$

where $\mathcal{Q}$ is a $p \times K$ matrix of linear inequality restrictions with rank p ($p \leq K$) and 0_p is a $(p \times 1)$-vector of zeros. The $\mathcal{Q}$ matrix can be set up to incorporate restrictions that either come from some *a priori* knowledge or from theory.

Given the normality result in Proposition 1, the test statistic is constructed by first solving the quadratic programming problem

$$\min_{\lambda} (\hat{\lambda} - \lambda)' \mathcal{Q}' (\mathcal{Q}\hat{V}(\hat{\lambda})\mathcal{Q}')^{-1} \mathcal{Q}(\hat{\lambda} - \lambda) \qquad \text{s.t.} \quad \mathcal{Q}\lambda \geq 0_p, \tag{34}$$

where $\hat{V}(\hat{\lambda})$ is a consistent estimator of $V(\hat{\lambda})$. Let $\tilde{\lambda}$ be the optimal solution of the problem in (34). The likelihood ratio test of the null hypothesis is

$$LR = T(\hat{\lambda} - \tilde{\lambda})'\mathcal{Q}'(\mathcal{Q}\hat{V}(\hat{\lambda})\mathcal{Q}')^{-1}\mathcal{Q}(\hat{\lambda} - \tilde{\lambda}). \tag{35}$$

For computational purposes, it is more convenient to consider the dual problem

$$\min_{\rho} \rho'\mathcal{Q}\hat{\lambda} + \frac{1}{2}\rho'(\mathcal{Q}\hat{V}(\hat{\lambda})\mathcal{Q}')\rho \qquad \text{s.t. } \rho \geq 0_p. \tag{36}$$

Let $\tilde{\rho}$ be the optimal solution of the problem in (36). The Kuhn-Tucker test of the null hypothesis is given by

$$KT = T\tilde{\rho}'(\mathcal{Q}\hat{V}(\hat{\lambda})\mathcal{Q}')\tilde{\rho}. \tag{37}$$

The objective functions of the primal and dual problems evaluated at the optimum $(\tilde{\lambda}, \tilde{\rho})$ are equal and we have that $LR = KT$.

To conduct statistical inference, we need to derive the asymptotic distribution of LR. Wolak (1989) shows that under $H_0 : \ \mathcal{Q}\lambda = 0_p$ (that is, the least favorable value of $\mathcal{Q}\lambda$ under the null hypothesis), LR has a weighted chi-squared distribution

$$LR \overset{A}{\sim} \sum_{i=0}^{p} w_i \left((\mathcal{Q}V(\hat{\lambda})\mathcal{Q}')^{-1} \right) X_i = \sum_{i=0}^{p} w_{p-i} \left(\mathcal{Q}V(\hat{\lambda})\mathcal{Q}' \right) X_i, \tag{38}$$

where the X_i's are independent χ^2 random variables with i degrees of freedom, $\chi_0^2 \equiv 0$, and the weights w_i sum up to one. To compute the p-value of LR, we replace $V(\hat{\lambda})$ with $\hat{V}(\hat{\lambda})$ in the weight functions.

3. Empirical analysis

In this section we evaluate whether several prominent multifactor models satisfy the sign restrictions imposed by the ICAPM. To obtain an *a priori* knowledge of the expected signs of the λ parameters, we first run multiple predictive time-series regressions of the changes in the investment opportunity set (proxied by the future expected return on the aggregate equity market) on the model-specific state variables. Next, we run cross-sectional regressions of average excess returns on the estimated covariances between the excess returns and the innovations in these state variables (i.e., the factors).

A multifactor model is said to satisfy the restrictions imposed by the ICAPM if the signs with which the model's state variables predict changes in the investment opportunity set coincide with the signs of the prices of covariance risk that their innovations earn in the cross-section. In addition, since the covariance price of market risk has a natural interpretation of relative risk aversion coefficient, we incorporate in our set of sign restrictions the constraint that the market premium should be positive.

In addition to the models considered in Maio and Santa-Clara (2012), we analyze the five-factor specification proposed by Fama and French (2015). More specifically, we estimate and test nine multifactor models. Four of these models are theory based and contain innovations in state variables that have often been used in the return predictability literature. The rest are empirically motivated models that have sometimes received an ICAPM interpretation in the asset-pricing literature.

The first of the theory motivated models is the specification of Hahn and Lee (2006), which extends the CAPM by including innovations in a term state variable and a default state variable. The multifactor model proposed by Petkova (2006) contains innovations in the dividend yield and in the risk-free rate in addition to the factors in the Hahn and Lee (2006) model. We also test an unrestricted version of the ICAPM specification of Campbell and Vuolteenaho (2004), which incorporates innovations in a price-to-earnings state variable, a term state variable, and a value spread state variable in addition to the market. The last theory motivated model is the multifactor model proposed by Koijen, Lustig, and Van Nieuwerburgh (2017), which includes, in addition to the market return, innovations in the term state variable and in the return-forecasting factor of Cochrane and Piazzesi (2005).

As for the empirically motivated models, the first model we consider is the Fama and French (1993) three-factor model, which extends the CAPM by including size and value in addition to the market. The Carhart (1997) four-factor model extends the Fama and French (1993) three-factor model by including a momentum factor. The Pastor and Stambaugh (2003) model extends the Fama and French (1993) three-factor model by including a liquidity factor. We also consider the five-factor model used by Fama and French (1993) to explain the expected returns on stocks and bonds. Their augmented model includes a term and a default factor in addition to the market, size, and value factors. Finally, we also estimate and test the five-factor model proposed by Fama and French (1993) which incorporates a profitability and an investment factor in addition to the classical three factors, namely market, size and value.

3.1. Predictive regressions for ICAPM state variables

In this section, we examine whether and with what sign the candidates state variables forecast changes in investment opportunities. The proxy for the investment opportunity set is the aggregate equity market and changes in investment opportunities are proxied by the monthly return on the value-weighted stock market index (from Kenneth French's website). The sample period is from July 1963 until December 2018. For each of the previously described models, we assess the joint forecasting power of the state variables by running multiple predictive time-series OLS regressions of the following form:

$$r_{t,t+q} = a_q + b_q z_t + u_{t,t+q}, \tag{39}$$

where $r_{t,t+q} = r_{t+1} + .. + r_{t+q}$ is the continuously compounded return over q periods, z_t is the set of candidate state variables corresponding to each model, and $u_{t,t+q}$ is a conditionally zero-mean forecasting error. The forecasting horizons q we consider are one, twelve, and sixty months. Here, our interest lies in the estimates of b_q and their associated t-statistics. This is indicative of whether a state variable forecasts positive or negative changes in future investment opportunities and of whether this effect is statistically significant.

We start by describing the state variables that will be used in the theory-based models. The predictive regressions for the Hahn and Lee (2006) model (HL) are given by

$$r_{t,t+q} = a_q + b_q TERM_t + c_q DEF_t + u_{t,t+q}, \tag{40}$$

where $TERM$ is slope of the Treasury yield curve, computed as the difference between the yields on ten-year and one-year Treasury bonds, and DEF is corporate bond default spread, computed as the difference between the yields on BAA- and AAA-rated corporate bonds. The yield data used for computing these factors are from the Federal Reserve Bank of St. Louis database (FRED).

For the Petkova (2006) model (P) we have

$$r_{t,t+q} = a_q + b_q TERM_t + c_q DEF_t + d_q DY_t + e_q RF_t + u_{t,t+q}, \tag{41}$$

where DY is the aggregate dividend-to-price ratio of the S&P Composite index, computed as the log ratio of annual dividends to the price level of the index (from Robert Shiller's website), and RF is the one-month Treasury bill rate (from Kenneth French's website).

In case of the Campbell and Vuolteenaho (2004) model (CV) we have

$$r_{t,t+q} = a_q + b_q TERM_t + c_q PE_t + d_q VS_t + u_{t,t+q}, \tag{42}$$

where PE is the aggregate price-to-earnings ratio of the S&P Composite index, computed as the log ratio of the price level of the index to a ten-year moving average of earnings (cyclically adjusted price-earnings) using data available on Robert Shiller's website, and VS is the value spread of Campbell and Vuolteenaho (2004), computed as the difference between the monthly log book-to-market ratios of the small high-book-to-market portfolio and the small low-book-to-market portfolio using data on the six portfolios sorted on size and book-to-market from Kenneth French's website.

Finally, for the Koijen, Lustig, and Van Nieuwerburgh (2017) model (KLVN), we formulate the predictive regression as

$$r_{t,t+q} = a_q + b_q TERM_t + c_q CP_t + u_{t,t+q}, \tag{43}$$

where CP is the Cochrane and Piazzesi (2005) factor, computed as the fitted value from a regression of the average (across maturities) excess bond return on a linear combination of forward rates using the Fama-Bliss data from CRSP.[3]

For the empirical specifications, the state variables are constructed as in Maio and Santa-Clara (2012). Specifically, in the case of the Fama and French (1993) three-factor model (FF3), the state variables corresponding to the size (SMB) and value (HML) factors are approximated using monthly market-to-book data on the six portfolios sorted on size and book-to-market (BM) from Kenneth French's website:

$$SMB^*_{FF3} = \frac{MB_{SL} + MB_{SM} + MB_{SH}}{3} - \frac{MB_{BL} + MB_{BM} + MB_{BH}}{3}, \tag{44}$$

$$HML^*_{FF3} = \frac{MB_{SH} + MB_{BH}}{2} - \frac{MB_{SL} + MB_{BL}}{2}, \tag{45}$$

where MB_{SL}, MB_{SM}, MB_{SH}, MB_{BL}, MB_{BM}, and MB_{BH} are the monthly market-to-book ratios of the small-low BM, small-medium BM, small-high BM, big-low BM, big-medium BM, and big-high BM portfolios. This approximation allows us to interpret SMB^*_{FF3} and HML^*_{FF3} as the state variables and the factors themselves as innovations in these state variables, that is, $SMB \simeq \Delta SMB^*_{FF3}$ and $HML \simeq \Delta HML^*_{FF3}$ (see Maio and Santa-Clara, 2012). Hence the

[3] For details on the construction of the CP factor see Cochrane and Piazzesi (2005).

predictive regression for FF3 is

$$r_{t,t+q} = a_q + b_q SMB^*_{FF3,t} + c_q HML^*_{FF3,t} + u_{t,t+q}. \tag{46}$$

For the Carhart (1997) model (C), we approximate the state variable associated with the momentum factor using cumulative sums of the factor returns over the previous 60 months:

$$CUMD_t = \sum_{s=t-59}^{t} UMD_s, \tag{47}$$

where UMD is the momentum factor (from Kenneth French's website). As pointed out by Maio and Santa-Clara (2012) and Cooper and Maio (2018), we use the 60 months cumulative sum because the total cumulative sum is close to being non-stationary and the momentum factors is approximated by the first difference in this constructed state variable $UMD \simeq \Delta CUMD$. Thus, the predictive regression takes the form

$$r_{t,t+q} = a_q + b_q SMB^*_{FF3,t} + c_q HML^*_{FF3,t} + d_q CUMD_t + u_{t,t+q}. \tag{48}$$

We adopt a similar approach for constructing the state variable associated with the liquidity factor in the Pastor and Stambaugh (2003) model (PS):

$$CL_t = \sum_{s=t-59}^{t} L_s, \tag{49}$$

where L is the non-traded liquidity factor from Lubos Pastor's website. The first difference in the state variable closely approximates the original factor, that is, $L \simeq \Delta CL$. The predictive regression is formulated as

$$r_{t,t+q} = a_q + b_q SMB^*_{FF3,t} + c_q HML^*_{FF3,t} + d_q CL_t + u_{t,t+q}. \tag{50}$$

The predictive regression for the Fama and French (1993) five-factor model (FFTD) that incorporates the bond-market factors $TERM$ and DEF is given by

$$r_{t,t+q} = a_q + b_q SMB^*_{FF3,t} + c_q HML^*_{FF3,t} + d_q TERM_t + e_q DEF_t + u_{t,t+q}, \tag{51}$$

where SMB^*_{FF3} and HML^*_{FF3} are the state variables defined in (44) and (45), respectively.

The state variables corresponding to the Fama and French (2015) five-factor model (FF5) are constructed using a similar approach to the one used for obtaining the FF3 state variables, except that we now use market-to-book data on three sets of portfolios instead of just one, namely the six portfolios sorted on size and book-to-market, the six portfolios sorted on size and operating profitability, as well as the six portfolios sorted on size and investment (from Kenneth French's website). More specifically, the state variables corresponding to the size (SMB), value (HML), profitability (RMW), and investment (CMA) factors are obtained by combining monthly market-to-book ratios across the relevant portfolios as follows:

$$SMB_{FF5}^{*} = \frac{SMB_{B/M} + SMB_{OP} + SMB_{INV}}{3}, \tag{52}$$

$$SMB_{B/M} = \frac{MB_{SL} + MB_{SM} + MB_{SH}}{3} - \frac{MB_{BL} + MB_{BM} + MB_{BH}}{3}, \tag{53}$$

$$SMB_{OP} = \frac{MB_{SW} + MB_{SM} + MB_{SR}}{3} - \frac{MB_{BW} + MB_{BM} + MB_{BR}}{3}, \tag{54}$$

$$SMB_{INV} = \frac{MB_{SC} + MB_{SM} + MB_{SA}}{3} - \frac{MB_{BC} + MB_{BM} + MB_{BA}}{3}, \tag{55}$$

$$HML_{FF5}^{*} = \frac{MB_{SH} + MB_{BH}}{2} - \frac{MB_{SL} + MB_{BL}}{2}, \tag{56}$$

$$RMW_{FF5}^{*} = \frac{MB_{SR} + MB_{BR}}{2} - \frac{MB_{SW} + MB_{BW}}{2}, \tag{57}$$

$$CMA_{FF5}^{*} = \frac{MB_{SC} + MB_{BC}}{2} - \frac{MB_{SA} + MB_{BA}}{2}, \tag{58}$$

where MB_{SW}, MB_{SM}, MB_{SR}, MB_{BW}, MB_{BM}, and MB_{BR} are the monthly market-to-book ratios of the small-weak profitability, small-medium profitability, small-robust profitability, big-weak profitability, big-medium profitability, and big-robust profitability portfolios, and MB_{SC}, MB_{SM}, MB_{SA}, MB_{BC}, MB_{BM}, and MB_{BA} are the monthly market-to-book ratios of the small-conservative investment, small-medium investment, small-aggressive investment, big-conservative investment, big-medium investment, and big-aggressive investment portfolios. As before, this approximation enables us to interpret the original factors as innovations in the state variables, that is, $SMB \simeq \Delta SMB_{FF5}^{*}$, $HML \simeq \Delta HML_{FF5}^{*}$, $RMW \simeq \Delta RMW_{FF5}^{*}$, and $CMA \simeq \Delta CMA_{FF5}^{*}$. Therefore, the predictive regression for the FF5 model is

$$r_{t,t+q} = a_q + b_q SMB_{FF5,t}^{*} + c_q HML_{FF5,t}^{*} + d_q RMW_{FF5,t}^{*} + e_q CMA_{FF5,t}^{*} + u_{t,t+q}. \tag{59}$$

In Table I we present estimation results for the multiple predictive regressions at horizons q of one, twelve, and sixty months. We report slope parameter estimates and associated t-ratios computed using Newey and West (1987) standard errors with q lags to correct for the serial correlation in the residuals induced by the overlapping cumulative returns.

In Panels A, B, C of Table I.1, we report the estimation results from the multiple predictive regressions corresponding to the theoretical models that have been explicitly proposed as ICAPM applications, at horizons of one, twelve, and sixty months, respectively. Several observations are in order. First, there seems to be stronger evidence of return predictability at longer horizons. For the one-month ahead predictive regressions (Panel A), only two out of eleven estimates are statistically significant at the 5% level, while for the sixty-month ahead predictive regressions five estimates are statistically significant at the 5% level (Panel C). The exceptions are the estimated coefficients on the $TERM$ state variable in the HL and KLVN models, on the DEF and RF state variables in the P model, on the VS state variable in the CV model, and the CP state variable in the KLVN model.

Second, the state variables do not predict future returns with the same signs across the different horizons considered. For instance, the RF state variable in the P model negatively affects future market returns in the one-month and twelve-month ahead predictive regressions (Panels A and B) but positively affects future market returns in the sixty-months ahead predictive regression (Panel C). However, these estimates are not statistically significant. Similarly, the VS state variable in the CV model has a negative and statistically significant estimated slope coefficient at the one-month horizon (Panel A), but this estimate becomes positive and statistically insignificant at the sixty-month horizon (Panel C).

In Panels A, B, and C of Table I.2, we report predictive regressions for the empirical specifications at horizons of one, twelve, and sixty months, respectively. The pattern of stronger return predictability at longer horizons seems to persist. At the one-month horizon (Panel A), none of the estimates is statistically significant at the 5% level. In contrast, at the sixty-month horizon (Panel C), the return predictability hypothesis receives support in seven out of sixteen instances at the 5% level.

Furthermore, we can observe the same issue of changing signs across predictive horizons. For example, the $CUMD$ state variable in the C model has a negative estimate in the one-month and twelve-month ahead predictive regressions (Panels A and B), but the slope estimate associated with it becomes positive in the sixty-month ahead predictive regression (Panel C). The CMA^*_{FF5} state variable behaves similarly. These estimates are not statistically significant at the 5% level though.

These results raise a number of questions. On the one hand, it is unclear what is the appropriate horizon over which the ability of the state variables to forecast future investment opportunities should be assessed. The horizon choice is somewhat arbitrary from an economic perspective, but from a statistical perspective the choice will naturally be driven by the availability of evidence in support of the predictability hypothesis. In our analysis of sign restrictions, we will rely on the sixty-month horizon, for which there is the strongest evidence of return predictability.

On the other hand, it is not entirely clear how to proceed when an estimate in the predictive regression is statistically insignificant, especially in light of the somewhat limited evidence in support of the predictability hypothesis discussed above. Since a statistically insignificant slope estimate is consistent with the true coefficient being either positive or negative, we believe that we should not impose a sign restriction on the corresponding price of covariance risk in this case. Surprisingly, previous studies have failed to take this aspect into account when evaluating the sign consistency of the considered models. Tests of sign consistency merely relied on an eye-balling exercise whereby the researcher simply compared the signs of the estimates in the time-series regressions with the signs of the estimates in the cross-sectional regressions, regardless of precision. As we will see later on, making inferences in the absence of statistical significance can strongly affect one's conclusions on the consistency of a multifactor model with the restrictions imposed by the ICAPM.

3.2. Multifactor models

In this section we examine the performance of several multifactor models in cross-sectional tests of asset-pricing models. Our main interest is in assessing whether and with what signs the innovations in the state variables are priced in the cross-section of equity returns. For each of the multifactor models considered, we estimate the prices of covariance risk by running two-pass cross-sectional regressions of average excess returns on the estimated factor covariances. The cross-sectional specification for a generic multifactor model is

$$\mu_R = V_{R,f}\lambda_f, \tag{60}$$

where μ_R are the expected excess returns on the test assets, $V_{R,f}$ are the covariances between the excess returns on the test assets and the innovations in the state variables, and λ_f are the prices of covariance risk.

The test assets returns used in the analysis are the monthly value-weighted returns on the 25

Fama-French size and book-to-market ranked portfolios, as well as the 25 Fama-French size and momentum ranked portfolios (from Kenneth French's website). The sample period runs from July 1963 until December 2018 (666 monthly observations). Following Maio and Santa-Clara (2012), we use first differences as proxies for the innovations in the state variables and use the notation Δ to indicate these first differences or changes.

In each of the nine models considered, the first factor is the excess market return (rm), which is proxied by the monthly return on the value-weighted stock market index in excess of the one-month Treasury bill rate (from Kenneth French's website). Hence, in the case of the Hahn and Lee (2006) model (HL), the cross-sectional specification is

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,\Delta term}\lambda_{\Delta term} + V_{R,\Delta def}\lambda_{\Delta def}, \tag{61}$$

where $\Delta term$ denotes the change in the slope of the Treasury yield curve and Δdef denotes the change in the corporate bond default spread.

For the ICAPM proposed by Petkova (2006) (P) we have

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,\Delta term}\lambda_{\Delta term} + V_{R,\Delta def}\lambda_{\Delta def} + +V_{R,\Delta dy}\lambda_{\Delta dy} + V_{R,\Delta rf}\lambda_{\Delta rf}, \tag{62}$$

where Δdy denotes changes in the aggregate dividend-to-price ratio and Δrf denotes changes in the one-month Treasury bill rate.

The Campbell and Vuolteenaho (2004) model (CV) takes the form

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,\Delta term}\lambda_{\Delta term} + V_{R,\Delta pe}\lambda_{\Delta pe} + V_{R,\Delta vs}\lambda_{\Delta vs}, \tag{63}$$

where Δpe denotes changes in the aggregate price-to-earnings ratio and Δvs denotes changes in the value spread of Campbell and Vuolteenaho (2004).

The Koijen, Lustig, and Van Nieuwerburgh (2017) model (KLVN) is

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,\Delta term}\lambda_{\Delta term} + V_{R,\Delta cp}\lambda_{\Delta cp}, \tag{64}$$

where Δcp denotes changes in the return-forecasting factor of Cochrane and Piazzesi (2005).

In the case of the Fama and French (1993) three-factor model (FF3) we have

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,smb}\lambda_{smb} + V_{R,hml}\lambda_{hml}, \tag{65}$$

where smb is the return difference between portfolios of stocks with small and big market capitalizations, and hml is the return difference between portfolios of stocks with high and low book-to-market ratios (from Kenneth French's website).

The cross-sectional specification for the Carhart (1997) four-factor model (C) is

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,smb}\lambda_{smb} + V_{R,hml}\lambda_{hml} + V_{R,umd}\lambda_{umd}, \tag{66}$$

where umd is return difference between portfolios of stocks with high and low prior returns (from Kenneth French's website).

For the Pastor and Stambaugh (2003) model (PS), we have

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,smb}\lambda_{smb} + V_{R,hml}\lambda_{hml} + V_{R,l}\lambda_l, \tag{67}$$

where l is the non-traded liquidity factor (from Lubos Pastor's website).

The Fama and French (1993) three-factor model augmented with the bond-market factors, the term spread and corporate default spread, (FFTD) is

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,smb}\lambda_{smb} + V_{R,hml}\lambda_{hml} + V_{R,\Delta term}\lambda_{\Delta term} + V_{R,\Delta def}\lambda_{\Delta def}. \tag{68}$$

Finally, the Fama and French (2015) five-factor model (FF5) is

$$\mu_R = V_{R,rm}\lambda_{rm} + V_{R,smb}\lambda_{smb} + V_{R,hml}\lambda_{hml} + V_{R,rmw}\lambda_{rmw} + V_{R,cma}\lambda_{cma}, \tag{69}$$

where rmw is the return difference between portfolios of stocks with robust and weak operating profitability, and cma is the return difference between portfolios of stocks with conservative and aggressive investment (from Kenneth French's website).

3.2.1. Sample cross-sectional R^2s of the models

In Table II, we report the sample cross-sectional R^2 ($\hat{\rho}^2$) for each model and investigate whether the model does a good job of explaining the cross-section of expected returns. We denote the p-value of a specification test of $H_0 : \rho^2 = 1$ by $p(\rho^2 = 1)$, and the p-value of a test of $H_0 : \rho^2 = 0$ by $p(\rho^2 = 0)$. Both tests are based on the asymptotic results in Section 2 for the sample cross-sectional R^2 statistic. We also provide an approximate F-test of model specification for comparison, denoted $\hat{Q}_c$. Next, we report the asymptotic standard error of the sample R^2, $\text{se}(\hat{\rho}^2)$, computed under the assumption of a misspecified model that provides some explanatory power i.e. $0 < \rho^2 < 1$. Finally, No. of para. is the number of parameters in each asset-pricing model.

The F-test is a generalized version of the cross-sectional regression test (CSRT) of Shanken (1985). It is based on a quadratic form in the model's deviations, $\hat{Q}_c = \hat{e}'\hat{V}(\hat{e})^{+}\hat{e}$, where $\hat{V}(\hat{e})$ is a consistent estimator of the asymptotic variance of the sample pricing errors and $\hat{V}(\hat{e})^{+}$ its pseudo-inverse. When the model is correctly specified (that is, $e = 0_N$ or $\rho^2 = 1$), we have $T\hat{Q}_c \overset{A}{\sim} \chi^2_{N-K-1}$. Following Shanken (1985), the reported p-value, $p(Q_c = 0)$, is for a transformation of $\hat{Q}_c$ that has an approximate F distribution: $\hat{Q}_c \overset{\text{app.}}{\sim} \left(\frac{N-K-1}{T-N+1}\right) F_{N-K-1,T-N+1}$.

In Panels A, B, and C of Table II.1, we provide results for the OLS, GLS, and WLS CSRs, respectively, for the case when the 25 size and book-to-market sorted portfolios are used as test assets. When estimation is done using OLS (Panel A), four models out of nine, namely FF3, PS, FFTD, and FF5, are rejected by the R^2 test, while the F-test indicates rejection of all but the HL and P models at the 5% level. The same four models are also rejected by the R^2 test when using WLS (Panel C) and the same significance level, while in this case the F-test rejects all but the P model. However, when estimation is made using GLS (Panel B), all the models except the P model are rejected at the 5% significance level by both the R^2 and the F-test. The null hypothesis that the model does not explain any of the variation in expected returns, ($H_0 : \rho^2 = 0$), is rejected at the 5% level for all the models and under all estimation methods.

Table II.2 is for the 25 size and momentum sorted portfolios. Based on the OLS R^2 (Panel A), four out of nine models, namely HL, CV, FF3 and C, are rejected at the 5% level, while the F-test indicates rejection of the same four models except the CV model. Using WLS (Panel C), all but the P, KLVN and FF5 models are rejected by the R^2 test, while the WLS F-test rejects four out of the nine models at the 5% level, namely the HL, CV, FF3 and C models. As for the test of $H_0 : \rho^2 = 0$, the null of no explanatory power is strongly rejected at the 1% level for all the models both in the OLS case (Panel A) and the WLS case (Panel C). When estimation is done using GLS (Panel B), the null hypothesis that the model is correctly specified ($H_0 : \rho^2 = 1$) is rejected at the 5% level for all the models by both the R^2 test and the F-test. Additionally, the hypothesis of no explanatory power $H_0 : \rho^2 = 0$ cannot be rejected at the 5% level in three instances, and indicates that this choice of test assets is particularly challenging for the HL, CV and KLVN models.

Although the results are sensitive to the criterion minimized in estimation as well as to the set of test assets used, there is widespread evidence of model misspecification. These are situations in which the use of misspecification-robust standard errors is likely to affect the outcomes of the parameter and multivariate inequality tests.

3.2.2. Properties of the $\boldsymbol{\lambda}$ estimates under correctly specified and potentially misspecified models

In this section, we follow what has been done in the literature and compare the signs of the time-series estimates with the signs of the cross-sectional estimates. In addition, we require the market price of covariance risk to be positive. We draw conclusions on sign consistency regardless of the statistical significance of the estimates.

In Table III, we report estimates of the price of covariance risk $\hat{\lambda}$ and associated t-ratios under correctly specified and potentially misspecified models. For correctly specified models, we give the t-ratio of Fama and MacBeth (1973), followed by that of Shanken (1992) and Jagannathan and Wang (1998), which account for estimation error in the covariances. Last, we report the t-ratio under potentially misspecified models, based on the results presented in Section 2. The various t-ratios are identified by subscripts fm, s, jw, and pm, respectively. Additionally, we also report the signs with which the underlying state variables predict future returns in the multiple time-series regressions (as a superscript in $\hat{\lambda}$, that is, $\hat{\lambda}^{(\pm)}$).

In Panels A, B, and C of Table III.1, we provide results for the OLS, GLS, and WLS CSRs, respectively, for the case when the 25 size and book-to-market sorted portfolios are used as test assets. When estimation is done using OLS (Table III.1, Panel A), only three models, namely FF3, C and FFTD, appear to be consistent with an ICAPM interpretation. However, none of the estimates with an inconsistent sign is statistically significant at the 5% level, except for the profitability (rmw) factor in the FF5 model. The estimate on the risk-free (rf) factor in the P model is also sign-inconsistent and statistically significant when Fama and and MacBeth (1973) standard errors are used in the estimation, but becomes insignificant when using EIV-corrected and misspecification-robust standard errors.

For GLS (Table III.1, Panel B), six out of nine models appear to be consistent with an ICAPM interpretation. Only three models, namely P, CV and FF5, have estimated prices of covariance risk whose signs are inconsistent with the signs of the estimated slopes from the predictive regressions. As for the statistical significance of the estimates with an inconsistent sign, most of them are not statistically significant after accounting for the estimation error in the covariances and for potential model misspecification. For instance, in case of the estimates on the market (rm) and dividend-yield (dy) factors in the P model, the Fama and MacBeth (1973) t-ratios indicate statistical significance at the 5% level but this is no longer the case if one considers the Shanken (1992), Jagannathan and Wang (1998) and misspecification-robust t-ratios. The only factor whose price of covariance risk is significant, as indicated by the set of all t-ratios, is the profitability (rmw) factor in the FF5 model.

The results for the WLS case (Table III.1, Panel C) indicate that the same six models, namely HL, KLVN, FF3, C, PS, and FFTD are consistent with the sign restrictions imposed by the ICAPM. Out of the models with price of covariance risk estimates whose signs do not coincide with the signs of the time-series estimates, only the FF5 model contains a coefficient that remains statistically significant at the 5% level using all sets of standard errors.

Table III.2 presents results for the 25 size and momentum sorted portfolios. In the OLS case (Table III.2, Panel A), only the C model appears to be consistent with an ICAPM interpretation as the signs of its price of covariance risk estimates coincide with the signs of the time-series estimates and it also satisfies the requirement that the market price of covariance risk is positive. The P, FF3 and PS models contain estimates with inconsistent signs that become insignificant when using EIV-corrected and misspecification-robust standard errors. On the other hand, the HL, CV, KLVN, FFTD and FF5 models contain estimates with an inconsistent sign that are statistically significant even after accounting for potential model misspecification.

For GLS (Table III.2, Panel B), six of the nine models have estimates that are not consistent with an ICAPM interpretation. The models that satisfy the ICAPM sign requirements are FF3, C, and PS. However, most of the coefficient estimates with an inconsistent sign are not statistically significant at the 5% level. Only the P and FF5 models contain a statistically significant estimate with an inconsistent sign as indicated by the set of all four t-ratios.

When WLS is used (Table III.2, Panel C), only the C model appears to satisfy the sign restrictions of the ICAPM. The HL, P, and PS models contain estimates for which the sign consistency is violated and which are statistically significant when using standard errors under correctly specified models but insignificant when using misspecification-robust standard errors. On the other hand, the CV, KLVN, FFTD, and FF5 models contain sign inconsistent estimates that are statistically significant even after controlling for model misspecification.

To summarize, if we simply compare the signs of the cross-sectional estimates with the signs of the time-series estimates and we require the market price of covariance risk to be positive, then the sign restrictions imposed by the ICAPM are satisfied in only 20 out of 54 cases. However, it should be noted that most cross-sectional estimates with an inconsistent sign are not statistically significant. In addition, accounting for model misspecification often makes a qualitative difference in terms of the conclusions reached.

3.2.3. Test of multiple sign restrictions

In this section, we employ the test of multiple sign restrictions discussed in Section 2.3 to assess whether the various models satisfy the time-series and cross-sectional restrictions imposed by the ICAPM. The use of this test will lead us to conclusions that are substantially different from the ones based on the comparative analysis of the previous section. The reason, in a nutshell, is that the test of multiple sign restrictions accounts for the estimation error in the parameters, for the joint significance of the estimates, and for potential model misspecification. Specifically, a coefficient estimate that is not statistically significant is consistent with the true coefficient being either positive or negative. This has two implications in the current setting. On the one hand, it is not clear whether a statistically insignificant cross-sectional estimate is consistent with the sign restriction obtained from the time series. On the other hand, and perhaps more importantly, it is not clear what sign restriction should be tested, if any, when the time-series estimate is statistically insignificant. In the following analysis, we will explore two cases: (i) when our *a priori* knowledge is simply based on the signs of the time-series estimates, regardless of their statistical significance, and (ii) when our *a priori* knowledge is only based on the signs of the time-series estimates that are statistically significant.

3.2.3.1 Imposing sign restrictions on all λ's

For a K-factor model, the test of multiple sign restrictions is a test of the null hypothesis $H_0 : \mathfrak{Q}\lambda \geq 0_K$ versus the alternative $H_1 : \lambda \in \Re^K$, where $\mathfrak{Q}$ is a $K \times K$ matrix of constraints with rank K. Specifically, when testing whether the coefficient associated with the k^{th} factor is positive (negative), we set the (k, k)-element of the Ω matrix equal to one (minus one), while the other elements in the k^{th} row of Ω are set equal to zero.

In Table IV, we report the values of the test statistic, LR, and associated p-values under correctly specified and potentially misspecified models. The specific form of $V(\hat{\lambda})$ in LR depends on whether the Fama and MacBeth (1973), Shanken (1992), Jagannathan and Wang (1998), or misspecification-robust asymptotic variances of the $\hat{\lambda}$'s are used (see Section 2). The corresponding likelihood ratio tests and their p-values are identified by the subscripts fm, s, jw, and pm, respectively.

In Panels A, B, and C of Table IV.1, we present results for the OLS, GLS, and WLS tests of multiple sign restrictions, respectively, for the case when the 25 size and book-to-market sorted portfolios are used as test assets. Under all estimation methods, FF5 is the only model for which the set of sign restrictions is systematically rejected at the 5% level by all four test statistics. For the other eight models, we are unable to reject the null hypothesis that the sign restrictions imposed by the ICAPM hold. These results are consistent with the analysis of the estimates of the prices of covariance risk in Table III.1. Out of all the estimates with an inconsistent sign, only the estimate associated with the profitability (rmw) factor in FF5 was statistically significant. Statistical precision of the λ estimates is clearly the key driver of the power of the test of sign restrictions.

It is worth noting that conducting inference under potential model misspecification often leads to qualitatively different conclusions. In general, the amount of evidence against the null hypothesis decreases when using misspecification-robust standard errors and we observe an increase in p-values. This closely matches the pattern of statistical significance of the λ estimates in the cross-sectional analysis of Table III.1. For instance, OLS and GLS test results (Table IV.1, Panels A and B, respectively) for the P model indicate that the sets of sign restrictions is rejected when the test statistics are based on the Fama and MacBeth (1973) standard errors, but not when inference is robustified against potential model misspecification. This is consistent with the pattern observed in Panels A and B of Table III.1, showing that the P model contains estimates with inconsistent signs that are statistically significant when using Fama and MacBeth (1973) standard errors but insignificant when using misspecification robust standard errors.

In Panels A, B, and C of Table IV.2, we employ the 25 size and momentum sorted portfolios as test assets. Only the FF5 model is systematically rejected under all estimation methods and by all test statistics at the 5% level. Additionally, in the OLS case (Table IV.2, Panel A) the consistency of the HL and CV models with the ICAPM implications is also rejected at the 5% level when misspecification-robust standard errors are used in the estimation. Interestingly, the misspecification-robust test statistic cannot reject the hypothesis of sign consistency in case of the KLVN and FFTD models despite the fact that these models contained a statistically significant estimate with an inconsistent sign as shown in Panel A of Table III.2. This is due to the fact that we are employing a test of joint restrictions across multiple factors; given that the other coefficient estimates in these models have the predicted sign and are statistically significant the evidence against the null is weakened. Similarly, when using GLS (Table IV.2, Panel B) and misspecification-robust standard errors the P model barely misses rejection with a p-value$_{pm}$ of 7%, which reflects the feature of imposing joint sign restrictions across multiple factors.

For WLS (Table IV.2, Panel C), the set of all four test statistics indicates that only the FF3 and C models satisfy the ICAPM restrictions, while the CV, KLVN, FFTD, and FF5 models do not satisfy the restrictions when considering a 5% confidence level. In line with the patterns of diminishing statistical significance shown in Panel A of Table III.3 the test indicates rejection of the HL, P and PS models when Fama and MacBeth (1973) standard errors are used in the estimation but this is no longer the case once misspecification-robust errors are employed.

Several observations emerge from the analysis. First, in 43 out of 54 cases, there is not enough evidence against the null of consistency with the ICAPM when using a 5% significance level and misspecification-robust standard errors.[4] Second, accounting for model misspecification can make a significant difference in terms of conclusions: when the test is implemented using the Fama and MacBeth (1973) standard errors then we observe 32 out of 54 instances in which there is not enough evidence to reject the null of sign consistency. Third, the amount of evidence against the null is driven by the statistical significance of the cross-sectional estimates. Finally, the statistical significance of the individual cross-sectional estimates is only indicative of the test results since the null hypothesis being tested is composite.

[4]We also explore the impact of autocorrelation on our results by using the automatic lag length selection procedure of Newey and West (1994) and reach similar conclusions. Specifically, failure to reject the null is now observed in 47 out of 54 cases, and the models that exhibit inconsistencies with the ICAPM at the 5% level using misspecification-robust estimation are the FF5 model for OLS and GLS using the 25 size and book-to-market sorted portfolios, the P model for OLS and GLS, the KLVN model for OLS and WLS, and the FFTD model for OLS using the 25 size and momentum sorted portfolios.

3.2.3.2 Imposing sign restrictions conditional on the state variables being robust predictors

As previously mentioned, it is not clear whether and what sign restrictions should be imposed when the time-series estimates are not statistically significant. The results presented in Table IV relate to the case in which the restrictions imposed are purely based on the signs of the time-series estimates, regardless of their statistical significance. We now explore the case in which sign restrictions are imposed conditional on the state variables being robust predictors. Specifically, if the state variable corresponding to the k^{th} factor in a K-factor model is not a robust predictor of future aggregate returns, we eliminate the corresponding row from the matrix of constraints Ω. Thus, the test of multiple sign restrictions is a test of the null hypothesis $H_0: \ \Omega\lambda \geq 0_p$ versus the alternative $H_1: \ \lambda \in \Re^K$, where Ω is the $p \times K$ matrix of constraints and $p \leq K$ is the number of restrictions being imposed.

In Table V, we report results by imposing sign restrictions only on the factors whose associated state variables have estimated time-series coefficients that are statistically significant at the 5% level, as shown in Panel C of Tables I.1 and I.2. More specifically, we maintain the sign restriction associated with the *term* factor in the P, CV and FFTD models, the def factor in the HL model, the *dy* factor in the P model, the *pe* factor in the CV model, the *smb* factor in the C model, the *hml* factor in the FF3, C, PS and FFTD models, and the *rmw* factor in the FF5 model. Additionally, we maintain the restriction that the market price of covariance risk is positive across all the models.

In Panels A, B, and C of Table V.1, we provide results for the OLS, GLS, and WLS tests of multiple sign restrictions, respectively, for the case when the 25 size and book-to-market sorted portfolios are used as test assets. The results are qualitatively similar to the baseline case presented in Table V.1. The only model that is systematically rejected across estimation methods and by the set of all four test statistics is the FF5 model. This accurately reflects the fact that for this model we kept the sign restriction on the price of covariance risk estimate associated with the profitability (*rmw*) factor, which as shown in Table III.1 was statistically significant and had an inconsistent sign. Generally, we observe a decrease in p-values relative to the baseline case when removing a restriction from coefficient estimates with a consistent sign (which is the case for the HL, CV, KLVN, FF3, C, PS, and FFTD models). However, when a restrictions is removed from a coefficient estimate with an inconsistent sign (which is the case for the P model) we observe an increase in p-values since the amount of evidence against the null of sign consistency decreases.

Table V.2 presents results for the 25 size and momentum sorted portfolios. Worth noting is the fact that the FF5 model is no longer systematically rejected, and we only observe rejection of the null when estimation is done using GLS (Table V.2, Panel B). Otherwise, the evidence against the null follows closely the pattern of statistical significance of the restricted sign cross-sectional coefficient, *rmw*, shown in Table III.2. Other notable differences relative to the baseline case are the HL model for OLS estimation, and the KLVN model for WLS estimation, both of which are now consistent with an ICAPM interpretation. Comparing Panel A of Table V.2 with Panel A of Table IV.2, we note that the HL now satisfies the ICAPM restrictions (p-value$_{pm}$ of 39% versus p-value$_{pm}$ of 4% in the baseline case). This is due to the removal of the sign restriction on the *term* factor which, as shown in Panel A of Table III.2, has a statistically significant cross-sectional estimate with an inconsistent sign. Similarly, the WLS p-value$_{pm}$ associated with the KLVN model increases beyond the point of rejecting the null (from 5% in the baseline case to 50%), which is due to the removal of the constraint on the *term* factor, whose estimate was marginally significant and had an inconsistent sign (Table III.2, Panel C).

Overall, we find that reducing the number of restrictions imposed on the model coefficients reduces the evidence against the null hypothesis of sign consistency relative to the baseline case whereby restrictions are imposed on all coefficients. The null hypothesis of consistency with an ICAPM interpretation is rejected in only 7 out of 54 cases when using a 5% significance level and misspecification-robust standard errors.[5]

Finally, we explore an alternative way of setting up the matrix of constraints, $\mathcal{Q}$, when the time-series estimates are statistically insignificant. We implement the new set of restrictions by setting equal to zero the price of covariance risk corresponding to a state variable that is not a robust predictor of future equity returns. Specifically, instead of eliminating the corresponding row from the matrix of constraints, we set each element in that row equal to zero. Table VI presents our results. The null of consistency with the ICAPM is rejected in only 1 of the 54 cases at the 5% level using misspecification-robust standard errors (namely the FFTD model when using WLS estimation and size and momentum sorted portfolios).[6]

[5]With a Newey and West (1994) automatic lag length selection adjustment, only in 3 out of 54 cases the misspecification-robust test statistics reject the null hypothesis of consistency with the ICAPM at the 5% level. Specifically, when using the size and momentum sorted portfolios as test assets the CV model for OLS estimation and the FFTD model for OLS and WLS estimation exhibit inconsistency with the ICAPM.

[6]When using a Newey and West (1994) automatic lag length selection adjustment all models are found to be

4. Conclusion

We develop a multivariate inequality framework for testing the consistency of multifactor asset-pricing models with the time-series and cross-sectional restrictions imposed by the ICAPM. Our test is based on results in the statistics literature due to Wolak (1987, 1989) and represents one of the first applications of Wolak's methods in empirical finance, alongside the ones in Kan, Robotti, and Shanken (2013) and Gospodinov, Kan, and Robotti (2013).

We apply our test to nine multifactor models using two different sets of portfolios as test assets and three alternative estimation schemes. We find little evidence of inconsistency of popular multifactor models with the restrictions imposed by the ICAPM. Our findings are at odds with the results in Maio and Santa-Clara (2012) who argue that most models do not satisfy the restrictions imposed by the ICAPM, but are in line with Boons (2016) and Barroso, Boons and Karehnke (2019) who use individual stock level evidence to show that most multifactor models are consistent with an ICAPM interpretation. Interestingly, using our testing framework, we are able to show that most multifactor models are consistent with the ICAPM restrictions even when portfolios instead of individual stocks are used in analysis.

In the extant literature the consistency of the models with the ICAPM restrictions is assessed by eye-balling the signs of the parameter estimates in the time-series and cross-sectional regressions. We go beyond this practice and propose a multivariate inequality test to assess the consistency of several multifactor models with the implications of the ICAPM. Specifically, our methodology accounts for the estimation error in the covariances and for the fact that the consistency of a multifactor model with the implications of the ICAPM should be evaluated using tests of joint sign restrictions across factors. We also take seriously the fact that asset-pricing models are only approximations to reality and are likely to be misspecified. Consistent with this view, we employ inference methods that are robust to model misspecification, in addition to the traditional methods that assume that the underlying model is correctly specified.

consistent with an ICAPM interpretation.

Appendix

Proof of Proposition 1: The proof relies on the fact that $\hat{\lambda}$ is a smooth function of $\hat{\mu}$ and $\hat{V}$. Therefore, once we have the asymptotic distribution of $\hat{\mu}$ and $\hat{V}$, we can use the delta method to obtain the asymptotic distribution of $\hat{\lambda}$. Let

$$\varphi = \begin{bmatrix} \mu \\ \mathrm{vec}(V) \end{bmatrix}, \qquad \hat{\varphi} = \begin{bmatrix} \hat{\mu} \\ \mathrm{vec}(\hat{V}) \end{bmatrix}. \tag{A.1}$$

We first note that $\hat{\mu}$ and $\hat{V}$ can be written as the generalized method of moments (GMM) estimator that uses the moment conditions $E[r_t] = 0_{(N+K)(N+K+1)}$, where

$$r_t = \begin{bmatrix} Y_t - \mu \\ \mathrm{vec}((Y_t - \mu)(Y_t - \mu)' - V) \end{bmatrix}. \tag{A.2}$$

Since this is an exactly identified system of moment conditions, it is straightforward to verify that under the assumption that Y_t is stationary and ergodic with finite fourth moment, we have[7]

$$\sqrt{T}(\hat{\varphi} - \varphi) \stackrel{A}{\sim} N(0_{(N+K)(N+K+1)}, S_0), \tag{A.3}$$

where

$$S_0 = \sum_{j=-\infty}^{\infty} E[r_t r_{t+j}']. \tag{A.4}$$

Using the delta method, the asymptotic distribution of $\hat{\lambda}$ under potentially misspecified models is given by

$$\sqrt{T}(\hat{\lambda} - \lambda) \quad \stackrel{A}{\sim} \quad N\left(0_K, \left[\frac{\partial \lambda}{\partial \varphi'}\right] S_0 \left[\frac{\partial \lambda}{\partial \varphi'}\right]'\right). \tag{A.5}$$

Define $K_{m,n}$ as a commutation matrix (see, for example, Magnus and Neudecker (1999)) such that $K_{m,n}\mathrm{vec}(A) = \mathrm{vec}(A')$ where A is an $m \times n$ matrix. In addition, we denote $K_{n,n}$ by K_n. Let Θ be an $N^2 \times N^2$ matrix such that $\mathrm{vec}(\Sigma_d) = \Theta\mathrm{vec}(\Sigma)$.[8]

(a) The partial derivatives of λ with respect to μ are given by

$$\frac{\partial \lambda}{\partial \mu_f'} = 0_{K\times K}, \tag{A.6}$$

$$\frac{\partial \lambda}{\partial \mu_R'} = A. \tag{A.7}$$

[7] Note that S_0 is a singular matrix as $\hat{V}$ is symmetric, so there are redundant elements in $\hat{\varphi}$. We could have written $\hat{\varphi}$ as $[\hat{\mu}', \mathrm{vech}(\hat{V})']'$, but the results are the same under both specifications.

[8] Specifically, Θ is a matrix with (i,i)-th element equal to one, where $i = 1, 1 + 1(N + 1), 1 + 2(N + 1), \ldots, 1 + (N - 1)(N + 1)$, and zeros elsewhere.

It is easy to obtain:

$$\frac{\partial \text{vec}(V_{R,f})}{\partial \text{vec}(V)'} = [I_K,\ 0_{K\times N}] \otimes [0_{N\times K},\ I_N]. \tag{A.8}$$

For the derivative of $\lambda = A\mu_R$ with respect to $\text{vec}(V)$, we use the product rule to obtain

$$\frac{\partial \lambda}{\partial \text{vec}(V)'} = (\mu_R' W V_{R,f} \otimes I_K)\frac{\partial \text{vec}(H)}{\partial \text{vec}(V)'} + (\mu_R' W \otimes H)\frac{\partial \text{vec}(V_{f,R})}{\partial \text{vec}(V)'}. \tag{A.9}$$

The second term is given by

$$(\mu_R' W \otimes H)\frac{\partial \text{vec}(V_{f,R})}{\partial \text{vec}(V)'} = [H,\ 0_{K\times N}] \otimes [0_K',\ \mu_R' W]. \tag{A.10}$$

For the first term, we use the chain rule to obtain

$$\begin{aligned}
& (\mu_R' W V_{R,f} \otimes I_K)\frac{\partial \text{vec}(H)}{\partial \text{vec}(V)'} \\
= \ & (\mu_R' W V_{R,f} \otimes I_K)\frac{\partial \text{vec}(H)}{\partial \text{vec}(H^{-1})'}\frac{\partial \text{vec}(H^{-1})}{\partial \text{vec}(V)'} \\
= \ & -(\mu_R' W V_{R,f} \otimes I_K)(H\otimes H)\left[(V_{f,R} W \otimes I_K)\frac{\partial \text{vec}(V_{f,R})}{\partial \text{vec}(V)'} + (I_K \otimes V_{f,R} W)\frac{\partial \text{vec}(V_{R,f})}{\partial \text{vec}(V)'}\right] \\
= \ & -(\lambda' \otimes H)\left\{([0_{K\times K},\ V_{f,R}W] \otimes [I_K,\ 0_{K\times N}])\, K_{N+K} + [I_K,\ 0_{K\times N}] \otimes [0_{K\times K},\ V_{f,R}W]\right\} \\
= \ & [H,\ 0_{K\times N}] \otimes [0_K',\ -\lambda' V_{f,R} W] + [-\lambda',\ 0_N'] \otimes [0_{K\times K},\ A].
\end{aligned} \tag{A.11}$$

Combining the two terms, we have

$$\frac{\partial \lambda}{\partial \text{vec}(V)'} = [H,\ 0_{K\times N}] \otimes \left[0_K',\ e'W\right] - \left[\lambda', 0_N'\right] \otimes [0_{K\times K},\ A]. \tag{A.12}$$

Using the expression of $\partial\lambda/\partial\varphi'$, we can simplify the asymptotic variance of $\hat{\lambda}$ to

$$V(\hat{\lambda}) = \sum_{j=-\infty}^{\infty} E[h_t(\varphi) h_{t+j}(\varphi)'], \tag{A.13}$$

where

$$\begin{aligned}
h_t(\varphi) &= \frac{\partial \lambda}{\partial \varphi'} r_t(\varphi) \\
&= A(R_t - \mu_R) + \text{vec}\left([0_K',\ e'W][(Y_t-\mu)(Y_t-\mu)' - V]\begin{bmatrix} H \\ 0_{N\times K}\end{bmatrix}\right) \\
&\quad - \text{vec}\left([0_{K\times K},\ A][(Y_t-\mu)(Y_t-\mu)' - V]\begin{bmatrix} \lambda \\ 0_N\end{bmatrix}\right) \\
&= (\lambda_t - \lambda) + H(f_t - \mu_f)u_t - A(R_t - \mu_R)(f_t-\mu_f)'\lambda + AV_{R,f}\lambda \\
&= (\lambda_t - \lambda) + AG_t\lambda + H(f_t - \mu_f)u_t.
\end{aligned} \tag{A.14}$$

This completes the proof of part (a).

(b) The partial derivatives of λ with respect to μ are the same as in the fixed weighting matrix case. For the derivative of λ with respect to $\text{vec}(V)$, we use the product rule to obtain

$$\frac{\partial\lambda}{\partial\text{vec}(V)'} = (\mu_R' V_R^{-1} V_{R,f} \otimes I_K)\frac{\partial\text{vec}(H)}{\partial\text{vec}(V)'} + (\mu_R' V_R^{-1} \otimes H)\frac{\partial\text{vec}(V_{f,R})}{\partial\text{vec}(V)'} + (\mu_R' \otimes H V_{f,R})\frac{\partial\text{vec}(V_R^{-1})}{\partial\text{vec}(V)'}. \tag{A.15}$$

The last two terms are given by

$$\begin{aligned}(\mu_R' V_R^{-1} \otimes H)\frac{\partial\text{vec}(V_{f,R})}{\partial\text{vec}(V)'} &= [H,\ 0_{K\times N}] \otimes [0_K',\ \mu_R' V_R^{-1}], & \text{(A.16)}\\ (\mu_R' \otimes H V_{f,R})\frac{\partial\text{vec}(V_R^{-1})}{\partial\text{vec}(V)'} &= -[0_K',\ \mu_R' V_R^{-1}] \otimes [0_{K\times K},\ A]. & \text{(A.17)}\end{aligned}$$

For the first term, we use the chain rule to obtain

$$\begin{aligned}&(\mu_R' V_R^{-1} V_{R,f} \otimes I_K)\frac{\partial\text{vec}(H)}{\partial\text{vec}(V)'}\\ =\ & (\mu_R' V_R^{-1} V_{R,f} \otimes I_K)\frac{\partial\text{vec}(H)}{\partial\text{vec}(H^{-1})'}\frac{\partial\text{vec}(H^{-1})}{\partial\text{vec}(V)'}\\ =\ & -(\mu_R' V_R^{-1} V_{R,f} \otimes I_K)(H \otimes H)\left[(V_{f,R} V_R^{-1} \otimes I_K)\frac{\partial\text{vec}(V_{f,R})}{\partial\text{vec}(V)'}\right.\\ & \left. + (V_{f,R} \otimes V_{f,R})\frac{\partial\text{vec}(V_R^{-1})}{\partial\text{vec}(V)'} + (I_K \otimes V_{f,R} V_R^{-1})\frac{\partial\text{vec}(V_{R,f})}{\partial\text{vec}(V)'}\right]\\ =\ & -(\lambda' \otimes H)\left\{\left([0_{K\times K},\ V_{f,R} V_R^{-1}] \otimes [I_K,\ 0_{K\times N}]\right) K_{N+K}\right.\\ & - [0_{K\times K},\ V_{f,R} V_R^{-1}] \otimes [0_{K\times K},\ V_{f,R} V_R^{-1}]\\ & \left. + [I_K,\ 0_{K\times N}] \otimes [0_{K\times K},\ V_{f,R} V_R^{-1}]\right\}\\ =\ & [H,\ 0_{K\times N}] \otimes [0_K',\ -\lambda' V_{f,R} V_R^{-1}]\\ & + [-\lambda',\ \lambda' V_{f,R} V_R^{-1}] \otimes [0_{K\times K},\ A]. \end{aligned} \tag{A.18}$$

Combining the three terms, we have

$$\frac{\partial\lambda}{\partial\text{vec}(V)'} = [H,\ 0_{K\times N}] \otimes \left[0_K',\ e' V_R^{-1}\right] - \left[\lambda',\ e' V_R^{-1}\right] \otimes [0_{K\times K},\ A]\,. \tag{A.19}$$

Using the expression of $\partial\lambda/\partial\varphi'$, we can simplify the asymptotic variance of $\hat{\lambda}$ to

$$V(\hat{\lambda}) = \sum_{j=-\infty}^{\infty} E[h_t(\varphi) h_{t+j}(\varphi)'], \tag{A.20}$$

where

$$
\begin{aligned}
h_t(\varphi) &= \frac{\partial\lambda}{\partial\varphi'}r_t(\varphi) \\
&= A(R_t-\mu_R)+\mathrm{vec}\left([0_K',\ e'V_R^{-1}][(Y_t-\mu)(Y_t-\mu)'-V]\begin{bmatrix} H \\ 0_{N\times K}\end{bmatrix}\right) \\
&\quad -\mathrm{vec}\left([0_{K\times K},\ A][(Y_t-\mu)(Y_t-\mu)'-V]\begin{bmatrix} \lambda \\ V_R^{-1}e\end{bmatrix}\right) \\
&= (\lambda_t-\lambda)+H(f_t-\mu_f)u_t-A(R_t-\mu_R)(f_t-\mu_f)'\lambda-A(R_t-\mu_R)u_t+AV_{R,f}\lambda \\
&= (\lambda_t-\lambda)+AG_t\lambda+H(f_t-\mu_f)u_t-(\lambda_t-\lambda)u_t. \qquad \text{(A.21)}
\end{aligned}
$$

This completes the proof of part (b).

(c) The partial derivatives of λ with respect to μ are the same as in the fixed weighting matrix case. For the derivative of λ with respect to $\mathrm{vec}(V)$, we use the product rule to obtain

$$
\frac{\partial\lambda}{\partial\mathrm{vec}(V)'}=(\mu_R'\Sigma_d^{-1}V_{R,f}\otimes I_K)\frac{\partial\mathrm{vec}(H)}{\partial\mathrm{vec}(V)'}+(\mu_R'\Sigma_d^{-1}\otimes H)\frac{\partial\mathrm{vec}(V_{f,R})}{\partial\mathrm{vec}(V)'}+(\mu_R'\otimes HV_{f,R})\frac{\partial\mathrm{vec}(\Sigma_d^{-1})}{\partial\mathrm{vec}(V)'}. \qquad \text{(A.22)}
$$

The last two terms are given by

$$
(\mu_R'\Sigma_d^{-1}\otimes H)\frac{\partial\mathrm{vec}(V_{f,R})}{\partial\mathrm{vec}(V)'} = [H,0_{K\times N}]\otimes[0_K',\ \mu_R'\Sigma_d^{-1}], \qquad \text{(A.23)}
$$

$$
(\mu_R'\otimes HV_{f,R})\frac{\partial\mathrm{vec}(\Sigma_d^{-1})}{\partial\mathrm{vec}(V)'} = -(\mu_R'\Sigma_d^{-1}\otimes A)\Theta([-\beta,\ I_N]\otimes[-\beta,\ I_N]). \qquad \text{(A.24)}
$$

For the first term, we use the chain rule to obtain

$$
\begin{aligned}
& (\mu_R' \Sigma_d^{-1} V_{R,f} \otimes I_K) \frac{\partial \mathrm{vec}(H)}{\partial \mathrm{vec}(V)'} \\
= \;& (\mu_R' \Sigma_d^{-1} V_{R,f} \otimes I_K) \frac{\partial \mathrm{vec}(H)}{\partial \mathrm{vec}(H^{-1})'} \frac{\partial \mathrm{vec}(H^{-1})}{\partial \mathrm{vec}(V)'} \\
= \;& -(\mu_R' \Sigma_d^{-1} V_{R,f} \otimes I_K)(H \otimes H) \Bigg[(V_{f,R} \Sigma_d^{-1} \otimes I_K) \frac{\partial \mathrm{vec}(V_{f,R})}{\partial \mathrm{vec}(V)'} \\
& + (V_{f,R} \otimes V_{f,R}) \frac{\partial \mathrm{vec}(\Sigma_d^{-1})}{\partial \mathrm{vec}(V)'} + (I_K \otimes V_{f,R} \Sigma_d^{-1}) \frac{\partial \mathrm{vec}(V_{R,f})}{\partial \mathrm{vec}(V)'} \Bigg] \\
= \;& -(\lambda' \otimes H) \big\{ \big([0_{K\times K},\ V_{f,R}\Sigma_d^{-1}] \otimes [I_K,\ 0_{K\times N}] \big) K_{N+K} \\
& - (V_{f,R}\Sigma_d^{-1} \otimes V_{f,R}\Sigma_d^{-1}) \Theta([-\beta,\ I_N] \otimes [-\beta,\ I_N]) \\
& + [I_K,\ 0_{K\times N}] \otimes [0_{K\times K},\ V_{f,R}\Sigma_d^{-1}] \big\} \\
= \;& [H,\ 0_{K\times N}] \otimes [0_K',\ -\lambda' V_{f,R}\Sigma_d^{-1}] \\
& + (\lambda' V_{f,R} \Sigma_d^{-1} \otimes A) \Theta([-\beta,\ I_N] \otimes [-\beta,\ I_N]) - [\lambda',\ 0_N'] \otimes [0_{K\times K},\ A]. \qquad \text{(A.25)}
\end{aligned}
$$

Combining the three terms, we have

$$
\begin{aligned}
\frac{\partial \lambda}{\partial \mathrm{vec}(V)'} = \;& [H,\ 0_{K\times N}] \otimes \left[0_K',\ e'\Sigma_d^{-1}\right] \\
& - [\lambda',\ 0_N'] \otimes [0_{K\times K},\ A] - (e'\Sigma_d^{-1} \otimes A)\Theta([-\beta,\ I_N] \otimes [-\beta,\ I_N]). \qquad \text{(A.26)}
\end{aligned}
$$

Using the expression of $\partial\lambda/\partial\varphi'$, we can simplify the asymptotic variance of $\hat{\lambda}$ to

$$
V(\hat{\lambda}) = \sum_{j=-\infty}^{\infty} E[h_t(\varphi) h_{t+j}(\varphi)'], \qquad \text{(A.27)}
$$

where

$$
\begin{aligned}
h_t(\varphi) = \;& \frac{\partial \lambda}{\partial \varphi'} r_t(\varphi) \\
= \;& A(R_t - \mu_R) + \mathrm{vec}\left([0_K',\ e'\Sigma_d^{-1}][(Y_t-\mu)(Y_t-\mu)' - V] \begin{bmatrix} H \\ 0_{N\times K} \end{bmatrix} \right) \\
& - \mathrm{vec}\left([0_{K\times K},\ A][(Y_t-\mu)(Y_t-\mu)' - V] \begin{bmatrix} \lambda \\ 0_N \end{bmatrix} \right) \\
& - (e'\Sigma_d^{-1} \otimes A)\Theta \mathrm{vec}\left([-\beta,\ I_N][(Y_t-\mu)(Y_t-\mu)' - V] \begin{bmatrix} -\beta' \\ I_N \end{bmatrix} \right) \\
= \;& (\lambda_t - \lambda) + H(f_t - \mu_f)u_t - A(R_t - \mu_R)(f_t - \mu_f)'\lambda \\
& + A V_{R,f}\lambda - (e'\Sigma_d^{-1} \otimes A)\Theta \mathrm{vec}(\epsilon_t \epsilon_t' - \Sigma) \\
= \;& (\lambda_t - \lambda) + A G_t \lambda + H(f_t - \mu_f)u_t - A\Psi_t \Sigma_d^{-1} e. \qquad \text{(A.28)}
\end{aligned}
$$

The second last equality follows from the first order condition $V_{f,R}\Sigma_d^{-1}e = 0_K$. This completes the proof of part (c).

Note that when the model is correctly specified, we have $e = 0_N$ and $u_t = 0$. In this case, we have

$$h_t(\varphi) = (\lambda_t - \lambda) + AG_t\lambda. \tag{A.29}$$

This completes the proof of Proposition 1.

Proof of Proposition 2:

(a) We first derive the asymptotic distribution of

$$T\hat{Q} = T(\hat{\mu}_R'\hat{W}\hat{\mu}_R - \hat{\mu}_R'\hat{W}\hat{\beta}(\hat{\beta}'\hat{W}\hat{\beta})^{-1}\hat{\beta}'\hat{W}\hat{\mu}_R) \tag{A.30}$$

under $H_0 : \rho^2 = 1$, where $\hat{W} \stackrel{\text{a.s.}}{\longrightarrow} W$ (this includes the known weighting matrix case as a special case). This can be accomplished by using the GMM results of Hansen (1982). Let $\theta = (\theta_1', \theta_2')'$, where $\theta_1 = (\alpha', \text{vec}(\beta)')'$ and $\theta_2 = \gamma$. Define

$$g_t(\theta) \equiv \begin{bmatrix} g_{1t}(\theta_1) \\ g_{2t}(\theta) \end{bmatrix} = \begin{bmatrix} l_t \otimes \epsilon_t \\ R_t - \beta\gamma \end{bmatrix}, \tag{A.31}$$

where $l_t = [1, f_t']'$ and $\epsilon_t = R_t - \alpha - \beta f_t$. When the model is correctly specified, we have $E[g_t(\theta)] = 0_{p+N}$, where $p = N(K+1)$. The sample moments of $g_t(\theta)$ are given by

$$\bar{g}_T(\theta) = \begin{bmatrix} \frac{1}{T}\sum_{t=1}^T g_{1t}(\theta_1) \\ \frac{1}{T}\sum_{t=1}^T g_{2t}(\theta) \end{bmatrix}. \tag{A.32}$$

Let $\hat{\theta} = (\hat{\theta}_1', \hat{\theta}_2')'$, where $\hat{\theta}_1 = (\hat{\alpha}', \text{vec}(\hat{\beta})')'$ is the OLS estimator of α and β, and

$$\hat{\theta}_2 = \hat{\gamma} = (\hat{\beta}'\hat{W}\hat{\beta})^{-1}\hat{\beta}'\hat{W}\hat{\mu}_R \tag{A.33}$$

is the second-pass CSR estimator of γ. Note that $\hat{\theta}$ is the solution to the following first-order condition

$$B_T\bar{g}_T(\theta) = 0_{p+K}, \tag{A.34}$$

where

$$B_T = \begin{bmatrix} I_p & 0_{p\times N} \\ 0_{K\times p} & \hat{\beta}'\hat{W} \end{bmatrix} \stackrel{\text{a.s.}}{\longrightarrow} \begin{bmatrix} I_p & 0_{p\times N} \\ 0_{K\times p} & \beta'W \end{bmatrix} \equiv B. \tag{A.35}$$

Writing

$$
\begin{aligned}
l_t \otimes \epsilon_t &= \text{vec}(\epsilon_t l_t') = (l_t \otimes I_N)\text{vec}(\epsilon_t), && \text{(A.36)}\\
\epsilon_t &= R_t - \alpha - \beta f_t = R_t - (l_t' \otimes I_N)\theta_1, && \text{(A.37)}\\
\beta\gamma &= (\gamma' \otimes I_N)\text{vec}(\beta), && \text{(A.38)}
\end{aligned}
$$

we have:

$$
\begin{aligned}
\frac{\partial g_{1t}(\theta_1)}{\partial \theta_1'} &= -l_t l_t' \otimes I_N, && \text{(A.39)}\\
\frac{\partial g_{1t}(\theta_1)}{\partial \theta_2'} &= 0_{p\times K}, && \text{(A.40)}\\
\frac{\partial g_{2t}(\theta)}{\partial \theta_1'} &= [0,\ -\gamma'] \otimes I_N, && \text{(A.41)}\\
\frac{\partial g_{2t}(\theta)}{\partial \theta_2'} &= -\beta. && \text{(A.42)}
\end{aligned}
$$

Let

$$
\begin{aligned}
D_T &= \frac{\partial \bar{g}_T(\theta)}{\partial \theta'}\\
&= \begin{bmatrix} -\left(\frac{1}{T}\sum_{t=1}^T l_t l_t'\right) \otimes I_N & 0_{p\times K} \\ [0,\ -\gamma'] \otimes I_N & -\beta \end{bmatrix}\\
&\xrightarrow{\text{a.s.}} \begin{bmatrix} -E[l_t l_t'] \otimes I_N & 0_{p\times K} \\ [0,\ -\gamma'] \otimes I_N & -\beta \end{bmatrix} \equiv D. && \text{(A.43)}
\end{aligned}
$$

Hansen (1982, Lemma 4.1) shows that when the model is correctly specified, we have:

$$
\sqrt{T}\bar{g}_T(\hat{\theta}) \overset{A}{\sim} N(0_{p+N}, [I_{p+N} - D(BD)^{-1}B]S[I_{p+N} - D(BD)^{-1}B]'), \quad \text{(A.44)}
$$

where

$$
S = \sum_{j=-\infty}^{\infty} E[g_t(\theta)g_{t+j}(\theta)']. \quad \text{(A.45)}
$$

Using the partitioned matrix inverse formula, it is easy to verify that

$$
E[l_t l_t']^{-1} = \begin{bmatrix} 1 + \mu_f' V_f^{-1}\mu_f & -\mu_f' V_f^{-1} \\ -V_f^{-1}\mu_f & V_f^{-1} \end{bmatrix}. \quad \text{(A.46)}
$$

It follows that

$$BD = \begin{bmatrix} -E[l_t l_t'] \otimes I_N & 0_{p\times K} \\ [0,\ -\gamma'] \otimes \beta' W & -H^{-1} \end{bmatrix}, \tag{A.47}$$

$$(BD)^{-1} = \begin{bmatrix} -E[l_t l_t']^{-1} \otimes I_N & 0_{p\times K} \\ [-\gamma' V_f^{-1}\mu_f,\ \gamma' V_f^{-1}] \otimes A & -H \end{bmatrix}, \tag{A.48}$$

$$D(BD)^{-1}B = \begin{bmatrix} I_p & 0_{p\times N} \\ [-\gamma' V_f^{-1}\mu_f,\ \gamma' V_f^{-1}] \otimes (I_N - \beta A) & -\beta A \end{bmatrix}, \tag{A.49}$$

$$I_N - D(BD)^{-1}B = \begin{bmatrix} 0_{p\times p} & 0_{p\times N} \\ [\gamma' V_f^{-1}\mu_f,\ -\gamma' V_f^{-1}] \otimes (I_N - \beta A) & I_N - \beta A \end{bmatrix}. \tag{A.50}$$

We now provide a simplification of the asymptotic distribution of $\bar{g}_{2T}(\hat{\theta})$. From (A.44), we have:

$$\sqrt{T}\bar{g}_{2T}(\hat{\theta}) \overset{A}{\sim} N(0_N, V_q), \tag{A.51}$$

where

$$V_q = \sum_{j=-\infty}^{\infty} E[q_t(\theta) q_{t+j}(\theta)'], \tag{A.52}$$

and

$$\begin{aligned} q_t(\theta) &= [0_{N\times p},\ I_N][I_{p+N} - D(BD)^{-1}B] g_t(\theta) \\ &= -(I_N - \beta A)\epsilon_t \gamma' V_f^{-1}(f_t - \mu_f) + (I_N - \beta A)(R_t - \beta\gamma) \\ &= (I_N - \beta A)[R_t - \epsilon_t \gamma' V_f^{-1}(f_t - \mu_f)] \\ &= (I_N - \beta A)\epsilon_t y_t \\ &= [I_N - \beta(\beta' W \beta)^{-1}\beta' W]\epsilon_t y_t \\ &= W^{-\frac{1}{2}}[I_N - W^{\frac{1}{2}}\beta(\beta' W \beta)^{-1}\beta' W^{\frac{1}{2}}] W^{\frac{1}{2}}\epsilon_t y_t \\ &= W^{-\frac{1}{2}}[I_N - W^{\frac{1}{2}} V_{R,f}(V_{f,R} W V_{R,f})^{-1} V_{f,R} W^{\frac{1}{2}}] W^{\frac{1}{2}}\epsilon_t y_t \\ &= W^{-\frac{1}{2}} P P' W^{\frac{1}{2}}\epsilon_t y_t, \end{aligned} \tag{A.53}$$

where $y_t = 1 - \lambda'(f_t - \mu_f) = 1 - \gamma' V_f^{-1}(f_t - \mu_f)$. The fourth equality follows from the fact that, under $H_0 : \rho^2 = 1$, $(I_N - \beta A)R_t = (I_N - \beta A)\epsilon_t$. With this expression of q_t, we can write V_q as

$$V_q = W^{-\frac{1}{2}} P P' W^{\frac{1}{2}} S W^{\frac{1}{2}} P P' W^{-\frac{1}{2}}, \tag{A.54}$$

where S is the asymptotic covariance matrix of $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\epsilon_t y_t$. Having derived the asymptotic distribution of $\bar{g}_{2T}(\hat{\theta})$, the asymptotic distribution of $\hat{Q}$ is given by

$$T\hat{Q} = T\bar{g}_{2T}(\hat{\theta})'\hat{W}\bar{g}_{2T}(\theta) \stackrel{A}{\sim} \sum_{j=1}^{N-K}\xi_j x_j, \tag{A.55}$$

where the x_j's are independent χ^2_1 random variables, and the ξ_j's are the $N-K$ nonzero eigenvalues of

$$W^{\frac{1}{2}}V_q W^{\frac{1}{2}} = PP'W^{\frac{1}{2}}SW^{\frac{1}{2}}PP'. \tag{A.56}$$

Equivalently, the ξ_j's are the eigenvalues of $P'W^{\frac{1}{2}}SW^{\frac{1}{2}}P$. Since $\hat{Q}_0 \xrightarrow{\text{a.s.}} Q_0 > 0$, we have:

$$T(\hat{\rho}^2 - 1) = -\frac{T\hat{Q}}{\hat{Q}_0} \stackrel{A}{\sim} -\sum_{j=1}^{N-K}\frac{\xi_j}{Q_0}x_j. \tag{A.57}$$

This completes the proof of part (a).

(b) The proof uses the same notation and delta method employed in Proposition 1 to obtain the asymptotic distribution of $\hat{\rho}^2$ as

$$\sqrt{T}(\hat{\rho}^2 - \rho^2) \stackrel{A}{\sim} N\left(0, \sum_{j=-\infty}^{\infty} E[n_t n_{t+j}]\right), \tag{A.58}$$

where

$$n_t = \frac{\partial\rho^2}{\partial\varphi'}r_t(\varphi). \tag{A.59}$$

Obtaining an explicit expression for n_t requires computing $\partial\rho^2/\partial\varphi'$. For the known weighting matrix case and the estimated GLS and WLS cases, we have

$$\frac{\partial\rho^2}{\partial\mu_f} = 0_K, \tag{A.60}$$

$$\frac{\partial\rho^2}{\partial\mu_R} = 2Q_0^{-1}W[(1-\rho^2)\mu_R - e]. \tag{A.61}$$

Equation (A.60) follows because ρ^2 does not depend on μ_f. For (A.61), using the first order condition $\beta'We = 0_K$ and letting $Q_0 = \mu_R'W\mu_R$, we have

$$\frac{\partial Q_0}{\partial\mu_R} = 2W\mu_R, \qquad \frac{\partial Q}{\partial\mu_R} = 2We. \tag{A.62}$$

It follows that

$$\frac{\partial\rho^2}{\partial\mu_R} = -Q_0^{-1}\frac{\partial Q}{\partial\mu_R} + Q_0^{-2}Q\frac{\partial Q_0}{\partial\mu_R} = -2Q_0^{-1}We + 2QQ_0^{-2}W\mu_R = 2Q_0^{-1}W[(1-\rho^2)\mu_R - e]. \tag{A.63}$$

The expression for $\partial\rho^2/\partial\text{vec}(V)'$, however, depends on whether we use a known W or an estimate of W, say $\hat{W}$, as the weighting matrix. We start with the known weighting matrix W case. Differentiating $Q = e'We$ with respect to $\text{vec}(V)$, we obtain:

$$\frac{\partial Q}{\partial\text{vec}(V)'} = 2e'W\frac{\partial(\mu_R - \beta\gamma)}{\partial\text{vec}(V)'} = -2e'W\left[(\gamma'\otimes I_N)\frac{\partial\text{vec}(\beta)}{\partial\text{vec}(V)'} + \beta\frac{\partial\gamma}{\partial\text{vec}(V)'}\right]. \tag{A.64}$$

Note that the second term vanishes because of the first order condition $\beta'We = 0_K$. Using

$$\frac{\partial\text{vec}(\beta)}{\partial\text{vec}(V)'} = [V_f^{-1},\ 0_{K\times N}]\otimes[-\beta,\ I_N]. \tag{A.65}$$

for the first term and the fact that $\beta'We = 0_K$ gives

$$\frac{\partial Q}{\partial\text{vec}(V)'} = -2e'W\left([\gamma'V_f^{-1},\ 0_N']\otimes[-\beta,\ I_N]\right) = -2\left([\gamma'V_f^{-1},\ 0_N']\otimes[0_K',\ e'W]\right). \tag{A.66}$$

Since $Q_0 = \mu_R'W\mu_R$ does not depend on V, we have:

$$\frac{\partial\rho^2}{\partial\text{vec}(V)'} = -Q_0^{-1}\frac{\partial Q}{\partial\text{vec}(V)'} = 2Q_0^{-1}\left[\gamma'V_f^{-1},\ 0_N'\right]\otimes\left[0_K',\ e'W\right]. \tag{A.67}$$

Therefore, for the known weighting matrix W case, n_t is given by

$$\begin{aligned} n_t &= \frac{\partial\rho^2}{\partial\varphi'}r_t(\varphi) \\ &= 2Q_0^{-1}[(1-\rho^2)\mu_R' - e']W(R_t - \mu_R) + 2Q_0^{-1}e'W(R_t-\mu_R)(f_t-\mu_f)'V_f^{-1}\gamma \\ &= 2Q_0^{-1}[-u_ty_t + (1-\rho^2)v_t]. \end{aligned} \tag{A.68}$$

We now turn to the $\hat{W} = \hat{V}_R^{-1}$ case. Differentiating $Q = e'V_R^{-1}e$ with respect to $\text{vec}(V)$, we obtain:

$$\begin{aligned} \frac{\partial Q}{\partial\text{vec}(V)'} &= 2e'V_R^{-1}\frac{\partial(\mu_R-\beta\gamma)}{\partial\text{vec}(V)'} + (e'\otimes e')\frac{\partial\text{vec}(V_R^{-1})}{\partial\text{vec}(V)'} \\ &= -2\left([\gamma'V_f^{-1},\ 0_N']\otimes[0_K',\ e'V_R^{-1}]\right) - (e'\otimes e')\left([0_{N\times K},\ V_R^{-1}]\otimes[0_{N\times K},\ V_R^{-1}]\right) \\ &= -[2\gamma'V_f^{-1},\ e'V_R^{-1}]\otimes[0_K',\ e'V_R^{-1}]. \end{aligned} \tag{A.69}$$

Similarly, we have:

$$\frac{\partial Q_0}{\partial\text{vec}(V)'} = -[0_K',\ \mu_R'V_R^{-1}]\otimes[0_K',\ \mu_R'V_R^{-1}]. \tag{A.70}$$

It follows that

$$\begin{aligned} \frac{\partial\rho^2}{\partial\text{vec}(V)'} &= -Q_0^{-1}\frac{\partial Q}{\partial\text{vec}(V)'} + Q_0^{-2}Q\frac{\partial Q_0}{\partial\text{vec}(V)'} \\ &= Q_0^{-1}\left[2\gamma'V_f^{-1},\ e'V_R^{-1}\right]\otimes\left[0_K',\ e'V_R^{-1}\right] \\ &\quad - Q_0^{-1}(1-\rho^2)\left[0_K',\ \mu_R'V_R^{-1}\right]\otimes\left[0_K',\ \mu_R'V_R^{-1}\right]. \end{aligned} \tag{A.71}$$

Therefore, we have:

$$\begin{aligned} n_t &= \frac{\partial \rho^2}{\partial \varphi'} r_t(\varphi) \\ &= 2Q_0^{-1}[(1-\rho^2)\mu_R' - e']V_R^{-1}(R_t - \mu_R) + Q_0^{-1}e'V_R^{-1}(R_t - \mu_R)[2\gamma' V_f^{-1}(f_t - \mu_f) \\ &\quad + e'V_R^{-1}(R_t - \mu_R)] - Q_0^{-1}(1-\rho^2)[\mu_R' V_R^{-1}(R_t - \mu_R)]^2 - Q_0^{-1}Q + Q_0^{-1}(1-\rho^2)Q_0 \\ &= Q_0^{-1}[u_t^2 - 2u_t y_t + (1-\rho^2)(2v_t - v_t^2)]. \end{aligned} \tag{A.72}$$

Finally, for the WLS case, we can use

$$\frac{\partial \mathrm{vec}(\Sigma_d^{-1})}{\partial \mathrm{vec}(V)'} = \frac{\partial \mathrm{vec}(\Sigma_d^{-1})}{\partial \mathrm{vec}(\Sigma_d)'} \frac{\partial \mathrm{vec}(\Sigma_d)}{\partial \mathrm{vec}(\Sigma)'} \frac{\partial \mathrm{vec}(\Sigma)}{\partial \mathrm{vec}(V)'} = -(\Sigma_d^{-1} \otimes \Sigma_d^{-1})\Theta([-\beta,\ I_N] \otimes [-\beta,\ I_N]). \tag{A.73}$$

and show that

$$\begin{aligned} \frac{\partial \rho^2}{\partial \mathrm{vec}(V)'} &= Q_0^{-1}\left\{\left[2\gamma' V_f^{-1},\ 0_N'\right] \otimes \left[0_K',\ e'\Sigma_d^{-1}\right] + \left(e'\Sigma_d^{-1} \otimes e'\Sigma_d^{-1}\right)\Theta\left([-\beta,\ I_N] \otimes [-\beta,\ I_N]\right)\right\} \\ &\quad - Q_0^{-1}(1-\rho^2)\left(\mu_R'\Sigma_d^{-1} \otimes \mu_R'\Sigma_d^{-1}\right)\Theta\left([-\beta,\ I_N] \otimes [-\beta,\ I_N]\right). \end{aligned} \tag{A.74}$$

It is then straightforward to obtain

$$\begin{aligned} n_t &= \frac{\partial \rho^2}{\partial \varphi'} r_t(\varphi) \\ &= 2Q_0^{-1}[(1-\rho^2)v_t - u_t] + 2Q_0^{-1}u_t\gamma' V_f^{-1}(f_t - \mu_f) + Q_0^{-1}e'\Sigma_d^{-1}\mathrm{Diag}(\epsilon_t\epsilon_t')\Sigma_d^{-1}e \\ &\quad - Q_0^{-1}(1-\rho^2)\mu_R'\Sigma_d^{-1}\mathrm{Diag}(\epsilon_t\epsilon_t')\Sigma_d^{-1}\mu_R - Q_0^{-1}Q + Q_0^{-1}(1-\rho^2)Q_0 \\ &= Q_0^{-1}\left[-2u_t y_t + e'\Gamma_t e + (1-\rho^2)(2v_t - \mu_R'\Gamma_t\mu_R)\right]. \end{aligned} \tag{A.75}$$

This completes the proof of part (b).

(c) We start by rewriting $Q_0 - Q$ as

$$\begin{aligned} Q_0 - Q &= \mu_R' W V_{R,f}(V_{f,R} W V_{R,f})^{-1} V_{f,R} W \mu_R \\ &= \lambda'(V_{f,R} W V_{R,f})\lambda. \end{aligned} \tag{A.76}$$

The matrix in the middle is positive definite because $V_{R,f}$ is assumed to be of full column rank. Therefore, the necessary and sufficient condition for $Q_0 = Q$ (that is, $\rho^2 = 0$) is $\lambda = 0_K$. Note that (A.76) also holds for its sample counterpart. As a consequence, we can write $\hat{\rho}^2$ as

$$\hat{\rho}^2 = 1 - \frac{\hat{Q}}{\hat{Q}_0} = \frac{\hat{Q}_0 - \hat{Q}}{\hat{Q}_0} = \frac{\hat{\lambda}'(\hat{V}_{f,R}\hat{W}\hat{V}_{R,f})\hat{\lambda}}{\hat{Q}_0}. \tag{A.77}$$

Under the null hypothesis $H_0 : \lambda = 0_K$, we have:

$$\sqrt{T}\hat{\lambda} \overset{A}{\sim} N(0_K, V(\hat{\lambda})), \tag{A.78}$$

where $V(\hat{\lambda})$ is the asymptotic variance of $\hat{\lambda}$ obtained under the potentially misspecified model. As $\hat{Q}_0 \xrightarrow{\text{a.s.}} Q_0 > 0$ and

$$\hat{V}_{f,R}\hat{W}\hat{V}_{R,f} \xrightarrow{\text{a.s.}} V_{f,R}WV_{R,f}, \tag{A.79}$$

it follows that

$$T\hat{\rho}^2 \overset{A}{\sim} \sum_{j=1}^{K} \frac{\xi_j}{Q_0} x_j, \tag{A.80}$$

where the x_j's are independent χ^2_1 random variables and the ξ_j's are the eigenvalues of

$$(V_{f,R}WV_{R,f})V(\hat{\lambda}). \tag{A.81}$$

This completes the proof of part (c).

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Table I.1

Multiple Predictive Regressions for Theory Motivated ICAPM State Variables

The table presents the estimation results of the multiple long-horizon predictive regressions corresponding to the models explicitly proposed as ICAPM applications. The forecasted variable is the monthly continuously compounded return on the value-weighted stock market index, at horizons q of 1, 12 and 60 months ahead. The forecasting variables are the current values of the term spread (TERM), default spread (DEF), market dividend yield (DY), one-month Treasury bill rate (RF), market price-earnings ratio (PE), value spread (VS), Cochrane-Piazzesi factor (CP). The original sample is from July 1963 to December 2018 but q observations are lost in each of the q-horizon regressions. We report parameter estimates and corresponding Newey-West t-ratios computed with q lags in parenthesis.

			Panel A: $q = 1$				
	TERM	DEF	DY	RF	PE	VS	CP
HL	0.15	0.51					
	(1.01)	(0.97)					
P	0.19	0.15	0.01	-0.64			
	(0.83)	(0.23)	(1.83)	(-0.47)			
CV	0.36				-0.00	-0.03	
	(2.19)				(-0.93)	(-2.40)	
KLVN	0.07						0.19
	(0.42)						(1.77)
			Panel B: $q = 12$				
	TERM	DEF	DY	RF	PE	VS	CP
HL	1.21	6.42					
	(0.85)	(2.11)					
P	1.43	2.39	0.14	-8.60			
	(0.85)	(0.72)	(2.29)	(-0.98)			
CV	2.78				-0.09	-0.14	
	(1.87)				(-1.82)	(-0.89)	
KLVN	1.02						0.95
	(0.61)						(0.94)
			Panel C: $q = 60$				
	TERM	DEF	DY	RF	PE	VS	CP
HL	5.50	26.75					
	(1.14)	(2.71)					
P	12.82	5.36	0.45	16.52			
	(2.68)	(0.55)	(4.96)	(0.69)			
CV	8.85				-0.52	0.22	
	(2.67)				(-7.32)	(1.33)	
KLVN	5.60						2.70
	(1.13)						(1.44)

Table I.2

Multiple Predictive Regressions for Empirically Motivated ICAPM State Variables

The table presents the estimation results of the multiple long-horizon predictive regressions corresponding to the empirical models that have been given a ICAPM interpretation. The forecasted variable is the monthly continuously compounded return on the value-weighted stock market index, at horizons q of 1, 12 and 60 months ahead. The forecasting variables are the current values of the size factor for the FF3 model (SMB^*_{FF3}), value factor for the FF3 model (HML^*_{FF3}), cumulative momentum factor (CUMD), cumulative liquidity factor (CL), term spread (TERM), default spread (DEF), size factor for the FF5 model (SMB^*_{FF5}), value factor for the FF5 model (HML^*_{FF5}), profitability factor for the FF5 model (RMW^*_{FF5}), investment factor for the FF5 model (CMA^*_{FF5}). The original sample is from July 1963 to December 2018 but q observations are lost in each of the q-horizon regressions. We report parameter estimates and corresponding Newey-West t-ratios computed with q lags in parenthesis.

	SMB^*_{FF3}	HML^*_{FF3}	CUMD	CL	TERM	DEF	SMB^*_{FF5}	HML^*_{FF5}	RMW^*_{FF5}	CMA^*_{FF5}
Panel A: $q = 1$										
FF3	0.01	0.00								
	(0.99)	(1.57)								
C	0.01	0.00	-0.01							
	(0.53)	(1.79)	(-1.33)							
PS	0.00	0.01		0.01						
	(0.07)	(1.70)		(1.04)						
FFTD	0.01	0.00			0.23	0.22				
	(0.88)	(1.38)			(1.45)	(0.34)				
FF5							0.00	-0.00	-0.00	0.01
							(0.43)	(-0.10)	(-0.45)	(1.67)
Panel B: $q = 12$										
FF3	0.17	0.02								
	(1.55)	(1.40)								
C	0.17	0.02	-0.01							
	(1.47)	(1.44)	(-0.15)							
PS	0.13	0.03		0.02						
	(0.97)	(1.24)		(0.48)						
FFTD	0.15	0.02			1.96	3.66				
	(1.44)	(1.25)			(1.44)	(1.10)				
FF5							0.06	-0.02	-0.06	0.07
							(1.04)	(-0.67)	(-0.90)	(0.83)
Panel C: $q = 60$										
FF3	0.39	0.14								
	(1.86)	(3.81)								
C	0.47	0.14	0.14							
	(2.41)	(3.48)	(0.77)							
PS	0.36	0.15		0.01						
	(1.07)	(2.31)		(0.09)						
FFTD	0.35	0.14			7.51	11.43				
	(1.73)	(4.76)			(2.01)	(1.00)				
FF5							0.12	-0.04	-0.40	-0.02
							(1.07)	(-0.48)	(-2.82)	(-0.14)

Table II.1
Sample Cross-Sectional R^2s and Specification Tests of the Models Using the 25 Size and Book-to-Market Portfolios as Test Assets

The table presents the sample cross-sectional R^2 ($\hat{\rho}^2$) and the generalized CSRT ($\hat{Q}_c$) of nine asset-pricing models. The models include the ICAPM specifications proposed by Hahn and Lee (2006) (HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). $p(\rho^2 = 1)$ is the p-value for the test of $H_0 : \rho^2 = 1$. $p(\rho^2 = 0)$ is the p-value for the test of $H_0 : \rho^2 = 0$. $\text{se}(\hat{\rho}^2)$ is the standard error of $\hat{\rho}^2$ under the assumption that $0 < \rho^2 < 1$. $p(Q_c = 0)$ is the p-value for the approximate F-test of $H_0 : Q_c = 0$. No. of para. is the number of parameters in the model.

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
$\hat{\rho}^2$	0.970	0.979	0.968	0.969	0.966	0.982	0.971	0.973	0.979
$p(\rho^2 = 1)$	0.231	0.678	0.069	0.199	0.000	0.214	0.000	0.007	0.000
$p(\rho^2 = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\text{se}(\hat{\rho}^2)$	0.020	0.018	0.021	0.023	0.020	0.015	0.017	0.017	0.013
$\hat{Q}_c$	0.049	0.025	0.052	0.054	0.144	0.052	0.099	0.061	0.092
$p(Q_c = 0)$	0.095	0.696	0.048	0.046	0.000	0.045	0.000	0.008	0.000
No. of para.	3	5	4	3	3	4	4	5	5
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
$\hat{\rho}^2$	0.288	0.433	0.310	0.355	0.212	0.491	0.246	0.313	0.336
$p(\rho^2 = 1)$	0.002	0.252	0.001	0.016	0.000	0.040	0.000	0.000	0.000
$p(\rho^2 = 0)$	0.005	0.014	0.019	0.002	0.000	0.000	0.003	0.008	0.000
$\text{se}(\hat{\rho}^2)$	0.150	0.209	0.146	0.155	0.074	0.152	0.092	0.129	0.095
$\hat{Q}_c$	0.081	0.037	0.080	0.061	0.145	0.062	0.115	0.088	0.096
$p(Q_c = 0)$	0.001	0.259	0.000	0.015	0.000	0.010	0.000	0.000	0.000
No. of para.	3	5	4	3	3	4	4	5	5
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
$\hat{\rho}^2$	0.979	0.983	0.979	0.980	0.974	0.986	0.976	0.979	0.982
$p(\rho^2 = 1)$	0.320	0.582	0.124	0.295	0.000	0.154	0.000	0.004	0.000
$p(\rho^2 = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\text{se}(\hat{\rho}^2)$	0.014	0.015	0.014	0.015	0.015	0.010	0.014	0.013	0.011
$\hat{Q}_c$	0.066	0.038	0.072	0.070	0.145	0.058	0.110	0.071	0.095
$p(Q_c = 0)$	0.007	0.239	0.002	0.004	0.000	0.018	0.000	0.001	0.000
No. of para.	3	5	4	3	3	4	4	5	5

Table II.2
Sample Cross-Sectional R^2s and Specification Tests of the Models Using the 25 Size and Momentum Portfolios as Test Assets

The table presents the sample cross-sectional R^2 ($\hat{\rho}^2$) and the generalized CSRT ($\hat{Q}_c$) of nine asset-pricing models. The models include the ICAPM specifications proposed by Hahn and Lee (2006) (HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). $p(\rho^2 = 1)$ is the p-value for the test of $H_0 : \rho^2 = 1$. $p(\rho^2 = 0)$ is the p-value for the test of $H_0 : \rho^2 = 0$. se($\hat{\rho}^2$) is the standard error of $\hat{\rho}^2$ under the assumption that $0 < \rho^2 < 1$. $p(Q_c = 0)$ is the p-value for the approximate F-test of $H_0 : Q_c = 0$. No. of para. is the number of parameters in the model.

				Panel A: OLS					
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
$\hat{\rho}^2$	0.856	0.928	0.880	0.910	0.787	0.971	0.851	0.925	0.979
$p(\rho^2 = 1)$	0.002	0.065	0.039	0.442	0.000	0.000	0.207	0.122	0.115
$p(\rho^2 = 0)$	0.001	0.001	0.001	0.001	0.001	0.000	0.003	0.001	0.000
se($\hat{\rho}^2$)	0.095	0.056	0.094	0.089	0.116	0.017	0.169	0.064	0.015
$\hat{Q}_c$	0.063	0.032	0.037	0.021	0.164	0.112	0.019	0.032	0.043
$p(Q_c = 0)$	0.011	0.420	0.304	0.917	0.000	0.000	0.928	0.437	0.117
No. of para.	3	5	4	3	3	4	4	5	5
				Panel B: GLS					
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
$\hat{\rho}^2$	0.116	0.347	0.087	0.145	0.127	0.369	0.164	0.202	0.476
$p(\rho^2 = 1)$	0.000	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.008
$p(\rho^2 = 0)$	0.089	0.009	0.314	0.173	0.003	0.000	0.022	0.028	0.000
se($\hat{\rho}^2$)	0.076	0.181	0.053	0.131	0.059	0.095	0.088	0.096	0.133
$\hat{Q}_c$	0.170	0.066	0.200	0.141	0.229	0.115	0.162	0.154	0.066
$p(Q_c = 0)$	0.000	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.003
No. of para.	3	5	4	3	3	4	4	5	5
				Panel C: WLS					
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
$\hat{\rho}^2$	0.907	0.942	0.912	0.933	0.917	0.978	0.933	0.952	0.989
$p(\rho^2 = 1)$	0.000	0.071	0.012	0.388	0.000	0.000	0.032	0.036	0.160
$p(\rho^2 = 0)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
se($\hat{\rho}^2$)	0.056	0.048	0.065	0.068	0.046	0.013	0.056	0.038	0.007
$\hat{Q}_c$	0.104	0.041	0.062	0.028	0.186	0.113	0.045	0.048	0.049
$p(Q_c = 0)$	0.000	0.157	0.010	0.690	0.000	0.000	0.117	0.062	0.057
No. of para.	3	5	4	3	3	4	4	5	5

Table III.1
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Book-to-Market Portfolios as Test Assets

The table presents the estimation results of nine asset-pricing models. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report parameter estimates $\hat{\lambda}$, the Fama and MacBeth (1973) t-ratio under correctly specified models (t-ratio$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) t-ratios under correctly specified models that account for the EIV problem (t-ratio$_s$ and t-ratio$_{jw}$, respectively), and our model misspecification-robust t-ratios (t-ratio$_{pm}$).

Panel A:OLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	-0.63	495.37	182.31	-7.25	339.23	725.36	-11.25	-1363.82
t-ratio$_{fm}$	(-0.55)	(4.66)	(0.65)	(-1.42)	(2.67)	(3.31)	(-1.13)	(-2.38)
t-ratio$_s$	(-0.32)	(2.70)	(0.38)	(-0.70)	(1.32)	(1.63)	(-0.55)	(-1.17)
t-ratio$_{jw}$	(-0.28)	(2.46)	(0.37)	(-0.71)	(1.17)	(1.71)	(-0.57)	(-1.12)
t-ratio$_{pm}$	(-0.29)	(2.53)	(0.35)	(-0.55)	(0.97)	(1.53)	(-0.40)	(-0.58)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	-0.65	485.08	-0.50	0.57	-0.29	430.61	29.08	3.26	1.74	6.56
t-ratio$_{fm}$	(-0.17)	(4.27)	(-0.07)	(0.10)	(-0.17)	(3.20)	(0.66)	(3.36)	(1.26)	(4.40)
t-ratio$_s$	(-0.10)	(2.56)	(-0.04)	(0.06)	(-0.11)	(1.78)	(0.73)	(3.27)	(1.23)	(4.25)
t-ratio$_{jw}$	(-0.10)	(2.27)	(-0.04)	(0.07)	(-0.10)	(1.88)	(0.41)	(3.01)	(1.25)	(4.25)
t-ratio$_{pm}$	(-0.10)	(2.39)	(-0.04)	(0.04)	(-0.11)	(2.03)	(0.39)	(3.01)	(1.25)	(4.23)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	7.70	1.14	14.19	21.07	-0.86	0.74	6.08	10.14
t-ratio$_{fm}$	(6.49)	(0.82)	(7.18)	(6.21)	(-0.48)	(0.50)	(4.12)	(2.55)
t-ratio$_s$	(4.80)	(0.62)	(5.28)	(4.60)	(-0.42)	(0.44)	(3.55)	(2.21)
t-ratio$_{jw}$	(3.75)	(0.48)	(4.09)	(3.53)	(-0.42)	(0.36)	(3.02)	(1.97)
t-ratio$_{pm}$	(3.55)	(0.49)	(4.18)	(2.91)	(-0.36)	(0.35)	(2.99)	(1.80)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	0.99	2.15	3.09	301.70	430.77	3.96	5.88	3.65	13.98	3.23
t-ratio$_{fm}$	(0.84)	(1.50)	(1.72)	(3.73)	(2.34)	(3.03)	(3.57)	(0.86)	(3.68)	(0.38)
t-ratio$_s$	(0.58)	(1.04)	(1.20)	(2.59)	(1.63)	(2.86)	(3.35)	(0.82)	(3.45)	(0.36)
t-ratio$_{jw}$	(0.48)	(1.07)	(1.12)	(2.38)	(1.77)	(2.72)	(3.33)	(0.81)	(3.04)	(0.37)
t-ratio$_{pm}$	(0.40)	(1.04)	(0.81)	(1.35)	(1.47)	(2.58)	(3.33)	(0.58)	(2.41)	(0.26)

Table III.1 (Continued)
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Book-to-Market Portfolios as Test Assets

Panel B:GLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	0.73	309.76	84.34	-10.97	279.96	443.78	-21.49	-593.66
t-ratio$_{fm}$	(0.72)	(5.09)	(0.70)	(-3.19)	(3.79)	(3.02)	(-3.23)	(-1.25)
t-ratio$_{s}$	(0.55)	(3.80)	(0.53)	(-1.92)	(2.28)	(1.82)	(-1.95)	(-0.75)
t-ratio$_{jw}$	(0.50)	(3.65)	(0.57)	(-1.93)	(2.21)	(1.88)	(-1.89)	(-0.79)
t-ratio$_{pm}$	(0.50)	(2.69)	(0.37)	(-1.29)	(1.52)	(1.28)	(-1.23)	(-0.46)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	-1.99	282.35	4.51	6.68	1.15	314.46	55.50	3.38	1.85	5.90
t-ratio$_{fm}$	(-0.83)	(4.49)	(1.12)	(1.58)	(1.14)	(5.17)	(3.00)	(3.55)	(1.37)	(4.03)
t-ratio$_{s}$	(-0.64)	(3.43)	(0.86)	(1.21)	(0.81)	(3.62)	(2.12)	(3.45)	(1.34)	(3.90)
t-ratio$_{jw}$	(-0.71)	(3.31)	(0.97)	(1.14)	(0.75)	(3.41)	(2.05)	(3.15)	(1.37)	(3.87)
t-ratio$_{pm}$	(-0.55)	(2.26)	(0.69)	(0.63)	(0.73)	(2.64)	(1.38)	(3.14)	(1.36)	(3.85)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	6.67	1.59	12.06	17.13	0.92	1.06	5.87	6.34
t-ratio$_{fm}$	(6.06)	(1.18)	(6.72)	(5.95)	(0.61)	(0.76)	(4.01)	(2.06)
t-ratio$_{s}$	(4.83)	(0.96)	(5.34)	(4.74)	(0.57)	(0.71)	(3.71)	(1.92)
t-ratio$_{jw}$	(3.85)	(0.82)	(4.26)	(4.05)	(0.54)	(0.66)	(3.30)	(1.70)
t-ratio$_{pm}$	(3.58)	(0.82)	(4.07)	(3.21)	(0.37)	(0.60)	(3.29)	(1.11)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	1.54	1.23	3.00	238.54	128.14	4.97	4.74	1.31	12.38	9.13
t-ratio$_{fm}$	(1.34)	(0.87)	(1.71)	(3.27)	(1.03)	(4.01)	(2.96)	(0.34)	(3.76)	(1.19)
t-ratio$_{s}$	(1.09)	(0.71)	(1.40)	(2.66)	(0.84)	(3.77)	(2.80)	(0.33)	(3.55)	(1.14)
t-ratio$_{jw}$	(1.00)	(0.69)	(1.37)	(2.52)	(0.91)	(3.60)	(2.80)	(0.31)	(2.91)	(1.17)
t-ratio$_{pm}$	(0.92)	(0.69)	(1.11)	(1.61)	(0.59)	(3.17)	(2.64)	(0.21)	(2.29)	(0.75)

Table III.1 (Continued)
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Book-to-Market Portfolios as Test Assets

Panel C:WLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	0.13	392.99	83.67	-2.43	257.46	425.95	-3.17	-1288.26
t-ratio$_{fm}$	(0.11)	(3.79)	(0.30)	(-0.43)	(2.02)	(2.09)	(-0.29)	(-2.18)
t-ratio$_{s}$	(0.08)	(2.55)	(0.20)	(-0.25)	(1.19)	(1.23)	(-0.17)	(-1.28)
t-ratio$_{jw}$	(0.07)	(2.43)	(0.19)	(-0.25)	(1.09)	(1.15)	(-0.16)	(-1.25)
t-ratio$_{pm}$	(0.07)	(2.44)	(0.18)	(-0.22)	(0.99)	(0.96)	(-0.14)	(-0.68)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	-0.41	348.48	1.05	3.81	0.72	329.01	26.14	3.39	1.67	6.08
t-ratio$_{fm}$	(-0.10)	(3.13)	(0.13)	(0.67)	(0.50)	(2.92)	(0.69)	(3.51)	(1.22)	(4.07)
t-ratio$_{s}$	(-0.07)	(2.24)	(0.09)	(0.48)	(0.37)	(2.14)	(0.51)	(3.41)	(1.19)	(3.94)
t-ratio$_{jw}$	(-0.07)	(2.12)	(0.09)	(0.51)	(0.35)	(2.03)	(0.48)	(3.13)	(1.21)	(3.93)
t-ratio$_{pm}$	(-0.07)	(2.06)	(0.09)	(0.33)	(0.36)	(2.00)	(0.47)	(3.14)	(1.21)	(3.92)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	7.16	1.26	12.69	18.58	0.22	0.92	5.62	7.73
t-ratio$_{fm}$	(6.31)	(0.92)	(6.67)	(5.90)	(0.13)	(0.63)	(3.81)	(2.15)
t-ratio$_{s}$	(4.89)	(0.73)	(5.16)	(4.58)	(0.12)	(0.58)	(3.45)	(1.95)
t-ratio$_{jw}$	(3.90)	(0.58)	(4.18)	(3.66)	(0.12)	(0.51)	(3.01)	(1.70)
t-ratio$_{pm}$	(3.61)	(0.59)	(4.01)	(2.82)	(0.08)	(0.46)	(2.97)	(1.14)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	0.55	0.57	1.82	343.94	69.40	4.14	5.50	2.56	13.05	4.76
t-ratio$_{fm}$	(0.46)	(0.40)	(1.02)	(4.29)	(0.41)	(3.21)	(3.37)	(0.62)	(3.63)	(0.57)
t-ratio$_{s}$	(0.33)	(0.29)	(0.73)	(3.07)	(0.30)	(3.04)	(3.19)	(0.59)	(3.43)	(0.54)
t-ratio$_{jw}$	(0.30)	(0.27)	(0.74)	(2.87)	(0.32)	(2.87)	(3.19)	(0.58)	(2.94)	(0.56)
t-ratio$_{pm}$	(0.26)	(0.28)	(0.54)	(1.65)	(0.23)	(2.66)	(3.14)	(0.41)	(2.35)	(0.40)

Table III.2
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Momentum Portfolios as Test Assets

The table presents the estimation results of nine asset-pricing models. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and momentum ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report parameter estimates $\hat{\lambda}$, the Fama and MacBeth (1973) t-ratio under correctly specified models (t-ratio$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) t-ratios under correctly specified models that account for the EIV problem (t-ratio$_s$ and t-ratio$_{jw}$, respectively), and our model misspecification-robust t-ratios (t-ratio$_{pm}$).

Panel A:OLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	6.88	-500.30	-341.54	-0.22	-607.32	51.95	-4.92	-3014.52
t-ratio$_{fm}$	(5.63)	(-4.36)	(-1.58)	(-0.07)	(-5.05)	(0.28)	(-0.80)	(-5.94)
t-ratio$_s$	(3.15)	(-2.44)	(-0.89)	(-0.03)	(-2.28)	(0.13)	(-0.36)	(-2.68)
t-ratio$_{jw}$	(2.27)	(-1.93)	(-0.82)	(-0.03)	(-1.67)	(0.12)	(-0.35)	(-2.09)
t-ratio$_{pm}$	(2.49)	(-2.26)	(-0.71)	(-0.02)	(-1.55)	(0.10)	(-0.19)	(-1.68)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	5.05	-719.79	6.01	21.23	10.94	-745.59	182.96	1.00	2.69	-8.33
t-ratio$_{fm}$	(1.37)	(-4.55)	(0.77)	(2.87)	(6.64)	(-5.48)	(4.62)	(0.91)	(1.84)	(-2.55)
t-ratio$_s$	(0.57)	(-1.89)	(0.32)	(1.20)	(2.20)	(-1.82)	(1.54)	(0.88)	(1.77)	(-2.45)
t-ratio$_{jw}$	(0.49)	(-1.59)	(0.30)	(1.00)	(1.68)	(-1.55)	(1.43)	(0.83)	(1.71)	(-2.43)
t-ratio$_{pm}$	(0.42)	(-2.21)	(0.26)	(0.71)	(2.09)	(-2.02)	(1.66)	(0.84)	(1.66)	(-1.86)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	5.59	1.17	12.44	6.67	-17.92	-1.83	-17.30	44.39
t-ratio$_{fm}$	(5.21)	(0.81)	(4.51)	(6.34)	(-5.56)	(-1.19)	(-4.15)	(7.03)
t-ratio$_s$	(4.75)	(0.75)	(4.13)	(5.73)	(-2.16)	(-0.46)	(-1.61)	(2.72)
t-ratio$_{jw}$	(3.81)	(0.70)	(3.80)	(3.92)	(-1.41)	(-0.37)	(-1.03)	(1.64)
t-ratio$_{pm}$	(3.82)	(0.69)	(3.68)	(3.97)	(-1.47)	(-0.29)	(-1.39)	(1.55)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	10.20	7.88	14.02	-870.28	398.16	10.46	8.18	-43.38	19.48	79.66
t-ratio$_{fm}$	(6.61)	(4.72)	(4.39)	(-5.97)	(2.30)	(6.58)	(3.61)	(-5.93)	(3.43)	(6.14)
t-ratio$_s$	(2.59)	(1.86)	(1.73)	(-2.34)	(0.91)	(4.33)	(2.40)	(-3.91)	(2.28)	(4.05)
t-ratio$_{jw}$	(1.73)	(1.68)	(1.47)	(-1.59)	(0.71)	(3.93)	(1.83)	(-3.24)	(1.57)	(3.70)
t-ratio$_{pm}$	(2.07)	(1.74)	(1.57)	(-2.01)	(0.50)	(3.24)	(1.67)	(-3.14)	(1.36)	(2.95)

Table III.2 (Continued)
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Momentum Portfolios as Test Assets

Panel B:GLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	3.76	-45.13	277.89	-2.33	-170.88	350.89	-6.76	-1875.44
t-ratio$_{fm}$	(3.76)	(-0.80)	(2.20)	(-1.01)	(-2.84)	(2.66)	(-1.57)	(-5.23)
t-ratio$_{s}$	(3.51)	(-0.76)	(2.06)	(-0.66)	(-1.86)	(1.74)	(-1.03)	(-3.41)
t-ratio$_{jw}$	(3.31)	(-0.72)	(2.04)	(-0.62)	(-1.56)	(1.66)	(-1.06)	(-3.23)
t-ratio$_{pm}$	(2.86)	(-0.47)	(1.14)	(-0.50)	(-1.23)	(1.19)	(-0.77)	(-2.03)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	1.65	-50.98	3.21	-1.57	4.19	-70.59	55.85	3.20	2.48	4.83
t-ratio$_{fm}$	(0.69)	(-0.89)	(0.77)	(-0.37)	(4.11)	(-1.24)	(3.02)	(3.20)	(1.76)	(2.08)
t-ratio$_{s}$	(0.68)	(-0.87)	(0.75)	(-0.36)	(3.45)	(-1.05)	(2.55)	(3.12)	(1.72)	(2.04)
t-ratio$_{jw}$	(0.66)	(-0.85)	(0.74)	(-0.34)	(3.07)	(-1.02)	(2.45)	(2.88)	(1.79)	(2.08)
t-ratio$_{pm}$	(0.38)	(-0.51)	(0.40)	(-0.18)	(2.44)	(-0.62)	(1.00)	(2.86)	(1.74)	(1.65)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	4.89	2.23	10.54	6.07	0.53	1.90	3.28	6.17
t-ratio$_{fm}$	(4.69)	(1.58)	(4.19)	(5.92)	(0.34)	(1.33)	(1.36)	(2.32)
t-ratio$_{s}$	(4.36)	(1.49)	(3.90)	(5.45)	(0.32)	(1.25)	(1.28)	(2.17)
t-ratio$_{jw}$	(3.58)	(1.50)	(3.69)	(3.90)	(0.30)	(1.22)	(1.08)	(1.85)
t-ratio$_{pm}$	(3.59)	(1.47)	(3.29)	(3.84)	(0.19)	(1.17)	(0.87)	(1.09)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	4.37	4.28	5.84	-119.76	366.15	8.05	8.84	-28.89	20.97	53.16
t-ratio$_{fm}$	(3.86)	(2.83)	(2.33)	(-1.97)	(2.71)	(5.57)	(4.27)	(-5.21)	(4.16)	(5.16)
t-ratio$_{s}$	(3.39)	(2.49)	(2.05)	(-1.74)	(2.39)	(4.30)	(3.31)	(-4.02)	(3.23)	(3.99)
t-ratio$_{jw}$	(2.98)	(2.64)	(2.11)	(-1.52)	(2.20)	(4.08)	(2.92)	(-3.94)	(2.61)	(3.93)
t-ratio$_{pm}$	(2.65)	(2.39)	(1.58)	(-1.06)	(1.32)	(3.34)	(2.60)	(-2.92)	(2.05)	(2.76)

Table III.2 (Continued)
Estimates and t-ratios of Prices of Covariance Risk Using the 25 Size and Momentum Portfolios as Test Assets

Panel C:WLS

	HL			P				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{dy}^{(+)}$	$\hat{\lambda}_{rf}^{(+)}$
Estimate	5.02	-277.13	-426.03	-1.14	-430.90	-228.62	-4.03	-2694.15
t-ratio$_{fm}$	(4.10)	(-2.61)	(-1.84)	(-0.37)	(-4.10)	(-1.12)	(-0.66)	(-5.58)
t-ratio$_{s}$	(2.93)	(-1.87)	(-1.32)	(-0.19)	(-2.09)	(-0.57)	(-0.34)	(-2.84)
t-ratio$_{jw}$	(2.21)	(-1.52)	(-1.27)	(-0.17)	(-1.56)	(-0.59)	(-0.32)	(-2.30)
t-ratio$_{pm}$	(2.12)	(-1.43)	(-1.05)	(-0.09)	(-0.98)	(-0.34)	(-0.12)	(-1.30)

	CV				KLVN			FF3		
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{pe}^{(-)}$	$\hat{\lambda}_{vs}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{cp}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$
Estimate	1.51	-521.52	9.83	12.59	9.49	-577.07	165.58	1.66	3.61	-5.50
t-ratio$_{fm}$	(0.40)	(-3.81)	(1.27)	(1.74)	(6.21)	(-4.71)	(4.39)	(1.59)	(2.49)	(-1.84)
t-ratio$_{s}$	(0.22)	(-2.05)	(0.69)	(0.94)	(2.40)	(-1.83)	(1.70)	(1.54)	(2.41)	(-1.79)
t-ratio$_{jw}$	(0.19)	(-1.74)	(0.64)	(0.85)	(1.90)	(-1.63)	(1.68)	(1.45)	(2.44)	(-1.84)
t-ratio$_{pm}$	(0.16)	(-2.29)	(0.57)	(0.50)	(2.77)	(-2.38)	(1.95)	(1.48)	(2.42)	(-1.65)

	C				PS			
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{umd}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{l}^{(+)}$
Estimate	5.22	1.84	11.97	6.40	-9.00	1.13	-10.84	24.63
t-ratio$_{fm}$	(4.90)	(1.28)	(4.41)	(6.14)	(-4.07)	(0.77)	(-3.12)	(6.01)
t-ratio$_{s}$	(4.50)	(1.20)	(4.06)	(5.59)	(-2.43)	(0.46)	(-1.87)	(3.57)
t-ratio$_{jw}$	(3.64)	(1.15)	(3.84)	(3.93)	(-1.69)	(0.40)	(-1.21)	(2.17)
t-ratio$_{pm}$	(3.64)	(1.14)	(3.74)	(3.90)	(-0.95)	(0.29)	(-1.35)	(1.17)

	FFTD					FF5				
	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(+)}$	$\hat{\lambda}_{term}^{(+)}$	$\hat{\lambda}_{def}^{(+)}$	$\hat{\lambda}_{rm}^{(+)}$	$\hat{\lambda}_{smb}^{(+)}$	$\hat{\lambda}_{hml}^{(-)}$	$\hat{\lambda}_{rmw}^{(-)}$	$\hat{\lambda}_{cma}^{(-)}$
Estimate	8.01	7.53	9.14	-656.02	334.33	9.09	9.77	-39.90	19.93	71.97
t-ratio$_{fm}$	(5.67)	(4.45)	(3.24)	(-5.35)	(1.79)	(5.38)	(4.14)	(-5.29)	(3.23)	(5.05)
t-ratio$_{s}$	(2.78)	(2.18)	(1.59)	(-2.63)	(0.88)	(3.72)	(2.87)	(-3.66)	(2.25)	(3.50)
t-ratio$_{jw}$	(1.88)	(2.03)	(1.45)	(-1.79)	(0.75)	(3.51)	(2.20)	(-3.29)	(1.57)	(3.47)
t-ratio$_{pm}$	(2.44)	(2.11)	(1.35)	(-2.72)	(0.49)	(3.03)	(2.07)	(-3.21)	(1.44)	(3.00)

Table IV.1
Multiple Sign Restriction Tests of the Models Using the 25 Size and Book-to-Market Portfolios as Test Assets

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where the restrictions imposed are based on the signs of the respective coefficients from the long-horizon predictive regressions, regardless of their statistical significance. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
LR_{fm}	0.30	8.12	0.03	0.03	0.00	0.00	0.23	0.00	27.89
p-value$_{fm}$	(0.67)	(0.02)	(0.77)	(0.89)	(0.87)	(0.88)	(0.87)	(0.97)	(0.00)
LR_s	0.10	1.96	0.01	0.01	0.00	0.00	0.18	0.00	24.17
p-value$_s$	(0.78)	(0.36)	(0.79)	(0.91)	(0.87)	(0.88)	(0.89)	(0.97)	(0.00)
LR_{jw}	0.08	1.99	0.01	0.01	0.00	0.00	0.17	0.00	17.53
p-value$_{jw}$	(0.79)	(0.37)	(0.79)	(0.91)	(0.86)	(0.89)	(0.88)	(0.97)	(0.00)
LR_{pm}	0.08	1.25	0.01	0.01	0.00	0.00	0.13	0.00	16.82
p-value$_{pm}$	(0.78)	(0.56)	(0.81)	(0.91)	(0.86)	(0.85)	(0.90)	(0.98)	(0.00)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	18.32	1.26	0.00	0.00	0.00	0.00	0.00	31.23
p-value$_{fm}$	(0.90)	(0.00)	(0.44)	(0.89)	(0.87)	(0.88)	(0.98)	(0.98)	(0.00)
LR_s	0.00	6.64	0.75	0.00	0.00	0.00	0.00	0.00	27.13
p-value$_{fm}$	(0.90)	(0.07)	(0.57)	(0.89)	(0.87)	(0.88)	(0.98)	(0.98)	(0.00)
LR_{jw}	0.00	7.32	0.93	0.00	0.00	0.00	0.00	0.00	22.03
p-value$_{jw}$	(0.89)	(0.05)	(0.51)	(0.90)	(0.86)	(0.89)	(0.97)	(0.98)	(0.00)
LR_{pm}	0.00	3.82	0.48	0.00	0.00	0.00	0.00	0.00	20.67
p-value$_{pm}$	(0.88)	(0.23)	(0.66)	(0.90)	(0.86)	(0.87)	(0.98)	(0.98)	(0.00)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	5.42	0.02	0.00	0.00	0.00	0.00	0.00	25.18
p-value$_{fm}$	(0.89)	(0.08)	(0.78)	(0.93)	(0.87)	(0.88)	(0.98)	(0.98)	(0.00)
LR_s	0.00	1.87	0.01	0.00	0.00	0.00	0.00	0.00	22.13
p-value$_{fm}$	(0.89)	(0.37)	(0.80)	(0.93)	(0.87)	(0.88)	(0.98)	(0.98)	(0.00)
LR_{jw}	0.00	1.87	0.01	0.00	0.00	0.00	0.00	0.00	16.34
p-value$_{jw}$	(0.88)	(0.38)	(0.79)	(0.92)	(0.86)	(0.89)	(0.97)	(0.97)	(0.00)
LR_{pm}	0.00	0.76	0.01	0.00	0.00	0.00	0.00	0.00	14.94
p-value$_{pm}$	(0.87)	(0.68)	(0.82)	(0.92)	(0.86)	(0.85)	(0.98)	(0.98)	(0.00)

Table IV.2
Multiple Sign Restriction Tests of the Models Using the 25 Size and Momentum Portfolios as Test Assets

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where the restrictions imposed are based on the signs of the respective coefficients from the long-horizon predictive regressions, regardless of their statistical significance. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
LR_{fm}	19.38	43.69	20.69	30.08	6.50	0.00	30.94	35.65	58.24
p-value$_{fm}$	(0.00)	(0.00)	(0.00)	(0.00)	(0.03)	(0.91)	(0.00)	(0.00)	(0.00)
LR_s	6.08	8.88	3.59	3.32	5.98	0.00	4.65	5.49	25.02
p-value$_s$	(0.04)	(0.02)	(0.09)	(0.16)	(0.04)	(0.90)	(0.15)	(0.13)	(0.00)
LR_{jw}	4.15	4.84	2.52	2.40	5.91	0.00	1.99	2.53	15.98
p-value$_{jw}$	(0.10)	(0.10)	(0.18)	(0.25)	(0.04)	(0.89)	(0.41)	(0.41)	(0.00)
LR_{pm}	5.92	6.13	4.88	4.07	3.48	0.00	3.31	4.05	13.71
p-value$_{pm}$	(0.04)	(0.07)	(0.05)	(0.11)	(0.12)	(0.89)	(0.24)	(0.22)	(0.01)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.65	34.16	1.15	1.54	0.00	0.00	0.00	3.87	54.50
p-value$_{fm}$	(0.56)	(0.00)	(0.39)	(0.36)	(0.87)	(0.91)	(0.97)	(0.24)	(0.00)
LR_s	0.58	14.44	1.10	1.11	0.00	0.00	0.00	3.02	31.80
p-value$_{fm}$	(0.58)	(0.00)	(0.40)	(0.45)	(0.86)	(0.90)	(0.97)	(0.33)	(0.00)
LR_{jw}	0.52	13.27	1.05	1.04	0.00	0.00	0.00	2.30	21.13
p-value$_{jw}$	(0.62)	(0.00)	(0.42)	(0.46)	(0.86)	(0.88)	(0.96)	(0.42)	(0.00)
LR_{pm}	0.22	6.23	0.34	0.39	0.00	0.00	0.00	1.12	15.04
p-value$_{pm}$	(0.73)	(0.07)	(0.58)	(0.67)	(0.85)	(0.89)	(0.98)	(0.65)	(0.00)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	10.12	36.17	14.52	22.15	3.40	0.00	16.71	28.67	47.89
p-value$_{fm}$	(0.01)	(0.00)	(0.00)	(0.00)	(0.13)	(0.91)	(0.00)	(0.00)	(0.00)
LR_s	5.21	9.35	4.22	3.33	3.20	0.00	5.96	6.90	22.57
p-value$_{fm}$	(0.06)	(0.01)	(0.06)	(0.16)	(0.14)	(0.90)	(0.08)	(0.07)	(0.00)
LR_{jw}	4.57	6.12	3.02	2.64	3.40	0.00	2.85	3.19	15.60
p-value$_{jw}$	(0.08)	(0.06)	(0.14)	(0.22)	(0.13)	(0.89)	(0.27)	(0.32)	(0.00)
LR_{pm}	4.68	6.79	5.22	5.66	2.74	0.00	1.98	7.40	13.66
p-value$_{pm}$	(0.08)	(0.06)	(0.04)	(0.05)	(0.18)	(0.89)	(0.42)	(0.05)	(0.01)

Table V.1
Robust Multiple Sign Restriction Tests of the Models Using the 25 Size and Book-to-Market Portfolios as Test Assets
No restrictions are imposed if the state variables are not robust predictors

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where we impose sign restrictions only if the respective coefficient estimates from the long-horizon predictive regressions are statistically significant; otherwise no restriction is imposed. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
LR_{fm}	0.30	2.02	0.03	0.03	0.00	0.00	0.23	0.00	13.52
p-value$_{fm}$	(0.50)	(0.18)	(0.62)	(0.43)	(0.71)	(0.85)	(0.52)	(0.93)	(0.00)
LR_s	0.10	0.49	0.01	0.01	0.00	0.00	0.18	0.00	11.94
p-value$_{fm}$	(0.61)	(0.45)	(0.65)	(0.46)	(0.71)	(0.85)	(0.55)	(0.93)	(0.00)
LR_{jw}	0.08	0.50	0.01	0.01	0.00	0.00	0.17	0.00	9.23
p-value$_{jw}$	(0.63)	(0.47)	(0.64)	(0.46)	(0.70)	(0.85)	(0.57)	(0.92)	(0.00)
LR_{pm}	0.08	0.30	0.01	0.01	0.00	0.00	0.13	0.00	5.81
p-value$_{pm}$	(0.62)	(0.58)	(0.66)	(0.46)	(0.71)	(0.81)	(0.58)	(0.95)	(0.02)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	10.58	1.26	0.00	0.00	0.00	0.00	0.00	14.15
p-value$_{fm}$	(0.72)	(0.00)	(0.29)	(0.50)	(0.72)	(0.85)	(0.73)	(0.93)	(0.00)
LR_s	0.00	3.85	0.75	0.00	0.00	0.00	0.00	0.00	12.58
p-value$_{fm}$	(0.72)	(0.08)	(0.39)	(0.50)	(0.71)	(0.85)	(0.73)	(0.93)	(0.00)
LR_{jw}	0.00	3.77	0.93	0.00	0.00	0.00	0.00	0.00	8.45
p-value$_{jw}$	(0.72)	(0.09)	(0.35)	(0.50)	(0.70)	(0.84)	(0.73)	(0.93)	(0.01)
LR_{pm}	0.00	1.66	0.48	0.00	0.00	0.00	0.00	0.00	5.23
p-value$_{pm}$	(0.71)	(0.27)	(0.43)	(0.50)	(0.70)	(0.82)	(0.73)	(0.94)	(0.03)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	0.18	0.02	0.00	0.00	0.00	0.00	0.00	13.17
p-value$_{fm}$	(0.72)	(0.59)	(0.65)	(0.50)	(0.71)	(0.85)	(0.73)	(0.93)	(0.00)
LR_s	0.00	0.06	0.01	0.00	0.00	0.00	0.00	0.00	11.76
p-value$_{fm}$	(0.72)	(0.67)	(0.66)	(0.50)	(0.71)	(0.85)	(0.73)	(0.93)	(0.00)
LR_{jw}	0.00	0.06	0.01	0.00	0.00	0.00	0.00	0.00	8.64
p-value$_{jw}$	(0.72)	(0.68)	(0.66)	(0.50)	(0.70)	(0.84)	(0.74)	(0.92)	(0.01)
LR_{pm}	0.00	0.05	0.01	0.00	0.00	0.00	0.00	0.00	5.50
p-value$_{pm}$	(0.72)	(0.71)	(0.67)	(0.50)	(0.70)	(0.82)	(0.73)	(0.94)	(0.02)

Table V.2
Robust Multiple Sign Restriction Tests of the Models Using the 25 Size and Momentum Portfolios as Test Assets
No restrictions are imposed if the state variables are not robust predictors

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where we impose sign restrictions only if the respective coefficient estimates from the long-horizon predictive regressions are statistically significant; otherwise no restriction is imposed. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
LR_{fm}	2.49	25.48	20.69	0.00	6.50	0.00	30.94	35.65	11.73
p-value$_{fm}$	(0.13)	(0.00)	(0.00)	(0.50)	(0.01)	(0.86)	(0.00)	(0.00)	(0.00)
LR_s	0.79	5.21	3.59	0.00	5.98	0.00	4.65	5.49	5.20
p-value$_{fm}$	(0.35)	(0.03)	(0.06)	(0.50)	(0.02)	(0.86)	(0.03)	(0.06)	(0.03)
LR_{jw}	0.67	2.78	2.52	0.00	5.91	0.00	1.99	2.53	2.45
p-value$_{jw}$	(0.37)	(0.12)	(0.13)	(0.50)	(0.02)	(0.85)	(0.12)	(0.25)	(0.13)
LR_{pm}	0.50	2.39	4.88	0.00	3.48	0.00	3.31	4.05	1.84
p-value$_{pm}$	(0.39)	(0.13)	(0.04)	(0.50)	(0.07)	(0.85)	(0.07)	(0.12)	(0.17)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	9.83	1.14	0.00	0.00	0.00	0.00	3.87	17.32
p-value$_{fm}$	(0.72)	(0.00)	(0.31)	(0.50)	(0.70)	(0.86)	(0.68)	(0.12)	(0.00)
LR_s	0.00	4.22	1.09	0.00	0.00	0.00	0.00	3.02	10.45
p-value$_{fm}$	(0.72)	(0.06)	(0.32)	(0.50)	(0.70)	(0.86)	(0.68)	(0.18)	(0.00)
LR_{jw}	0.00	3.37	1.04	0.00	0.00	0.00	0.00	2.30	6.82
p-value$_{jw}$	(0.71)	(0.10)	(0.33)	(0.50)	(0.69)	(0.84)	(0.69)	(0.25)	(0.01)
LR_{pm}	0.00	2.04	0.34	0.00	0.00	0.00	0.00	1.12	4.20
p-value$_{pm}$	(0.70)	(0.20)	(0.49)	(0.50)	(0.69)	(0.84)	(0.66)	(0.47)	(0.05)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	3.37	16.82	14.52	0.00	3.40	0.00	16.71	28.67	10.42
p-value$_{fm}$	(0.07)	(0.00)	(0.00)	(0.50)	(0.07)	(0.86)	(0.00)	(0.00)	(0.00)
LR_s	1.75	4.38	4.22	0.00	3.20	0.00	5.96	6.90	5.05
p-value$_{fm}$	(0.18)	(0.05)	(0.05)	(0.50)	(0.07)	(0.86)	(0.01)	(0.03)	(0.03)
LR_{jw}	1.61	2.45	3.02	0.00	3.40	0.00	2.85	3.19	2.46
p-value$_{jw}$	(0.19)	(0.15)	(0.10)	(0.50)	(0.07)	(0.84)	(0.08)	(0.18)	(0.13)
LR_{pm}	1.11	0.96	5.22	0.00	2.74	0.00	1.98	7.40	2.06
p-value$_{pm}$	(0.24)	(0.26)	(0.03)	(0.50)	(0.10)	(0.84)	(0.15)	(0.02)	(0.15)

Table VI.1
Robust Multiple Sign Restriction Tests of the Models Using the 25 Size and Book-to-Market Portfolios as Test Assets
Zero restrictions are imposed if the state variables are not robust predictors

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where we impose sign restrictions only if the respective coefficient estimates from the long-horizon predictive regressions are statistically significant; otherwise a zero restriction is imposed. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

Panel A: OLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.30	2.02	0.03	0.03	0.00	0.00	0.23	0.00	13.52
p-value$_{fm}$	(0.66)	(0.43)	(0.80)	(0.82)	(0.86)	(0.93)	(0.79)	(0.98)	(0.00)
LR_s	0.10	0.49	0.01	0.01	0.00	0.00	0.18	0.00	11.94
p-value$_{fm}$	(0.76)	(0.76)	(0.82)	(0.84)	(0.86)	(0.92)	(0.82)	(0.98)	(0.01)
LR_{jw}	0.08	0.50	0.01	0.01	0.00	0.00	0.17	0.00	9.23
p-value$_{jw}$	(0.78)	(0.76)	(0.81)	(0.84)	(0.85)	(0.93)	(0.82)	(0.98)	(0.02)
LR_{pm}	0.08	0.30	0.01	0.01	0.00	0.00	0.13	0.00	5.81
p-value$_{pm}$	(0.77)	(0.83)	(0.82)	(0.84)	(0.85)	(0.91)	(0.84)	(0.99)	(0.10)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	10.58	1.26	0.00	0.00	0.00	0.00	0.00	14.15
p-value$_{fm}$	(0.86)	(0.01)	(0.45)	(0.88)	(0.86)	(0.93)	(0.93)	(0.98)	(0.00)
LR_s	0.00	3.85	0.75	0.00	0.00	0.00	0.00	0.00	12.58
p-value$_{fm}$	(0.86)	(0.22)	(0.57)	(0.88)	(0.86)	(0.92)	(0.93)	(0.98)	(0.01)
LR_{jw}	0.00	3.77	0.93	0.00	0.00	0.00	0.00	0.00	8.45
p-value$_{jw}$	(0.86)	(0.23)	(0.52)	(0.88)	(0.85)	(0.92)	(0.93)	(0.98)	(0.03)
LR_{pm}	0.00	1.66	0.48	0.00	0.00	0.00	0.00	0.00	5.23
p-value$_{pm}$	(0.85)	(0.51)	(0.63)	(0.88)	(0.85)	(0.91)	(0.93)	(0.99)	(0.13)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	0.18	0.02	0.00	0.00	0.00	0.00	0.00	13.17
p-value$_{fm}$	(0.86)	(0.85)	(0.82)	(0.88)	(0.86)	(0.92)	(0.93)	(0.98)	(0.00)
LR_s	0.00	0.06	0.01	0.00	0.00	0.00	0.00	0.00	11.76
p-value$_{fm}$	(0.86)	(0.90)	(0.83)	(0.88)	(0.85)	(0.92)	(0.93)	(0.98)	(0.01)
LR_{jw}	0.00	0.06	0.01	0.00	0.00	0.00	0.00	0.00	8.64
p-value$_{jw}$	(0.86)	(0.90)	(0.82)	(0.88)	(0.85)	(0.92)	(0.93)	(0.98)	(0.03)
LR_{pm}	0.00	0.05	0.01	0.00	0.00	0.00	0.00	0.00	5.50
p-value$_{pm}$	(0.86)	(0.91)	(0.83)	(0.88)	(0.85)	(0.91)	(0.93)	(0.99)	(0.11)

Table VI.2
Robust Multiple Sign Restriction Tests of the Models Using the 25 Size and Momentum Portfolios as Test Assets
Zero restrictions are imposed if the state variables are not robust predictors

The table presents the results of the multiple sign restriction test of nine asset-pricing models for the case where we impose sign restrictions only if the respective coefficient estimates from the long-horizon predictive regressions are statistically significant; otherwise a zero restriction is imposed. This is a likelihood ratio test of the null hypothesis that the models satisfy the sign restrictions placed by the ICAPM $H_0 : \mathcal{Q}\lambda \geq 0_K$. The models include the ICAPM specifications proposed by Hahn and Lee (2006)(HL), Petkova (2006) (P), Campbell and Vuolteenaho (2004) (CV), Koijen, Lustig, and Van Nieuwerburgh (2017) (KLVN), the Fama and French (1993) three-factor model (FF3), the Carhart (1997) model (C), the Pastor and Stambaugh (2003) model (PS), the Fama and French (1993) three-factor model augmented by TERM and DEF (FFTD), and the Fama and French (2015) five-factor model (FF5). The models are estimated using monthly excess returns on the 25 Fama-French size and book-to-market ranked portfolios. The data are from July 1963 to December 2018 (666 observations). We report the values of the likelihood ratio statistics and corresponding p-values obtained using the Fama and MacBeth (1973) variances under correctly specified models (LR_{fm} and p-value$_{fm}$), the Shanken (1992) and the Jagannathan and Wang (1998) variances under correctly specified models that account for the EIV problem (LR_s and p-value$_s$, and LR_{jw} and p-value$_{jw}$, respectively), and our model misspecification-robust variances (LR_{pm} and p-value$_{jw}$).

	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
Panel A: OLS									
LR_{fm}	2.49	25.48	20.69	0.00	6.50	0.00	30.94	35.65	11.73
p-value$_{fm}$	(0.21)	(0.00)	(0.00)	(0.88)	(0.03)	(0.93)	(0.00)	(0.00)	(0.01)
LR_s	0.79	5.21	3.59	0.00	5.98	0.00	4.65	5.49	5.20
p-value$_{fm}$	(0.50)	(0.12)	(0.15)	(0.88)	(0.04)	(0.93)	(0.11)	(0.12)	(0.13)
LR_{jw}	0.67	2.78	2.52	0.00	5.91	0.00	1.99	2.53	2.45
p-value$_{jw}$	(0.52)	(0.33)	(0.25)	(0.88)	(0.04)	(0.92)	(0.35)	(0.39)	(0.39)
LR_{pm}	0.50	2.39	4.88	0.00	3.48	0.00	3.31	4.05	1.84
p-value$_{pm}$	(0.56)	(0.37)	(0.09)	(0.88)	(0.12)	(0.92)	(0.21)	(0.21)	(0.49)
Panel B: GLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	0.00	9.83	1.14	0.00	0.00	0.00	0.00	3.87	17.32
p-value$_{fm}$	(0.86)	(0.02)	(0.48)	(0.88)	(0.85)	(0.93)	(0.92)	(0.23)	(0.00)
LR_s	0.00	4.22	1.09	0.00	0.00	0.00	0.00	3.02	10.45
p-value$_{fm}$	(0.86)	(0.19)	(0.49)	(0.88)	(0.85)	(0.93)	(0.92)	(0.31)	(0.01)
LR_{jw}	0.00	3.37	1.04	0.00	0.00	0.00	0.00	2.30	6.82
p-value$_{jw}$	(0.86)	(0.26)	(0.50)	(0.88)	(0.85)	(0.92)	(0.92)	(0.42)	(0.07)
LR_{pm}	0.00	2.04	0.34	0.00	0.00	0.00	0.00	1.12	4.20
p-value$_{pm}$	(0.85)	(0.44)	(0.68)	(0.88)	(0.85)	(0.92)	(0.92)	(0.65)	(0.20)
Panel C: WLS									
	HL	P	CV	KLVN	FF3	C	PS	FFTD	FF5
LR_{fm}	3.37	16.82	14.52	0.00	3.40	0.00	16.71	28.67	10.42
p-value$_{fm}$	(0.13)	(0.00)	(0.00)	(0.88)	(0.13)	(0.93)	(0.00)	(0.00)	(0.01)
LR_s	1.75	4.38	4.22	0.00	3.20	0.00	5.96	6.90	5.05
p-value$_{fm}$	(0.30)	(0.17)	(0.12)	(0.88)	(0.14)	(0.93)	(0.06)	(0.06)	(0.14)
LR_{jw}	1.61	2.45	3.02	0.00	3.40	0.00	2.85	3.19	2.46
p-value$_{jw}$	(0.31)	(0.37)	(0.20)	(0.88)	(0.13)	(0.92)	(0.25)	(0.30)	(0.39)
LR_{pm}	1.11	0.96	5.22	0.00	2.74	0.00	1.98	7.40	2.06
p-value$_{pm}$	(0.40)	(0.62)	(0.08)	(0.88)	(0.18)	(0.92)	(0.36)	(0.05)	(0.45)